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Artigo

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Dynamic stand growth model for Norway spruce forests based on long-term experiments in Germany

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Abstract In the present study a dynamic stand growth model for Norway spruce (*Picea abies* (L.) Karst.) forests was developed from data of 99 research plots, which had been measured between one and seventeen times in northwestern and southern Germany. In this model, the initial stand conditions at any point in time are defined by three state variables (dominant height, basal area and number of trees per hectare), and they are used to estimate stand volume for a given projection age. The model uses three transition functions to project the stand state variables at any particular time and a state function to estimate the stand volume. Two of these transition functions were derived by expanding a base model and considering two site-specific parameters and the third one was derived considering one site-specific parameter. All the functions were fitted simultaneously using full information maximum likelihood and the base-age-invariant dummy variables method. The values of the critical error statistic for stand volume and the RMSE relative to the mean stand volume indicated that the overall model provides satisfactory predictions for time intervals up to 25 years.

Keywords GADA equations, simultaneous fitting, *Picea abies*.

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Introduction

Norway spruce (*Picea abies* (L.) Karst) is a European tree species with a wide natural range, extending from the Pyrenees, Alps and Balkans northwards through Germany to Scandinavia and eastwards to western Russia. Due to its good timber quality and excellent performance on a variety of growing sites, Norway spruce is one of the most widely planted and economically important tree species in Europe. The species has a long history of cultivation in Central Europe and has been planted very intensively since the middle of the 19th century. According to the second German National Forest Inventory, conducted during the period 2001–2002 (BWI, 2002), the total area covered by spruce amounts to 2,978,203 ha, of which 1,446,651 are privately owned.

Growth models are vitally important for forest management planning (Clutter et al., 1983). Numerous studies have been published dealing with the growth of Norway spruce. Among the first comprehensive investigations based on long-term field experiments in Germany are the yield tables (e.g Assmann & Franz, 1963). Studies about the response of Norway spruce to different thinning types and intensities were published by Abetz (1975) while a number of authors dealt with the effects of forest density and site (e.g. Schübeler, 1997).

During the 1990's a number of individual tree models were developed for spruce (e.g. Sterba, 1995 or Pretzsch, 2002). The study by Windhager (1999), which represents one of very few detailed critical evaluations of growth models in central Europe, includes a qualitative and a quantitative analysis of existing individual tree growth models for spruce and beech. The quantitative evaluation shows the substantial errors of the growth predictions for individual trees, without reference however to their accumulated effect on the stand level. One of the results of the qualitative analysis is the finding that model complexity does not necessarily increase the accuracy of the predictions. Furthermore, errors in individual-tree models may be magnified by inaccurate inventory data but remain less affected by simpler models such as stand models. In summary, stand models are preferable in many situations because they represent a good compromise between generality and accuracy of the estimates (Gadow, 1996; García, 2003).

The aim of this study is to develop a dynamic stand model for Norway spruce forests in Germany, based on an extensive published dataset. Our objective with this contribution is to present a new approach to modelling the growth of managed spruce forests, reducing model complexity and at the same time increasing the accuracy of prediction. In addition, the error of prediction for different projection intervals is specified.

Material and methods

Data

This study is based on an extensive set of long-term experimental plots from north-western and southern Germany. The growth data from north-western Germany were published by Schübeler (1997), those from southern Germany by Röhle (1995). Both authors provide a detailed description of the data which were previously used in the study of Vilčko (2005).

The dataset includes 99 plots, measured between 1 and 17 times. Altogether 911 observation intervals were available for this study including age (t, years), dominant height (H, m), stand basal area (G, m^2/ha), number of stems per hectare (N) and total stand volume (V, m^3/ha).

The relevant summary statistics of the main stand variables calculated for all the plots and the ratios of basal area and number of trees removed in thinnings are shown in Table 1.

Model structure

The proposed stand growth model is based on the state-space approach described by García (1994), which makes use of state variables to characterize the system at any point in time, and transition functions to project the state variables to future states.

The stand conditions at any point in time were defined by three state variables (dominant height, number of trees per hectare and stand basal area) and the model include three transition functions. The outputs of these three transition functions provide the inputs for a static function, which predicts the total stand volume for a given projection age.

In the present study, the transition function for reduction in tree number was derived using the algebraic difference approach (ADA) developed by Bailey & Clutter (1974) and considering one site-specific parameter and the transition functions for dominant height and stand basal area growth were derived using its generalization known as GADA (Cieszewski & Bailey, 2000) with two parameters being site-specific. The base-age invariant method of the dummy variables described in Cieszewski et al. (2000) was used to estimate the parameters of the transition functions.

Because of the longitudinal nature of the data sets used, correlation between the residuals within the same plot may be expected. To overcome any possible autocorrelation, we modelled the error terms using a continuous time autoregressive error structure (CAR(x)). To account for k-order autocorrelation, a CAR(x) model form that expands the error terms in the following way can be used:

$$e_{ij} = \sum_{k=1}^{k=x} I_k \rho_k^{t_{ij}-t_{ij-k}} e_{ij-k} + \varepsilon_{ij} \qquad (1)$$

where e_{ij} is the j^{th} ordinary residual on the i^{th} sample plot, e_{ij-k} is the j-k^{th} ordinary residual on the i^{th} sample plot, $I_k = 1$ for $j > k$ and it is zero for $j < k$, ρ_k is the k-order autoregressive parameter to be estimated, and $t_{ij} - t_{ij-k}$ is the time that separates the j^{th} from the j–k^{th} observations within each individual. In such cases ε_{ij} now includes the error terms under conditions of independence.

The growth model fitted involved two steps. In a first step, several dynamic equations derived from different base-models and different static functions for stand volume estimation were analysed separately and the best equation for each stand variable was selected by application of the criteria described in the following section. In the second step, the selected equations were fitted simultaneously using full information maximum likelihood (FIML) to minimize the total sum of squared errors of the system and to take into account the correlation between residuals of all the equations and the simultaneous equation bias. Since different amounts of data for the state variables and for volume were available, an associated dummy variable was included to specify the number of data and to allow use of all the data when fitting the equations simultaneously.

Stand variable	Mean	Minimum	Maximum	Standard Deviation
Stand age (t, years)	74.67	22.00	138.00	24.30
Dominant height (H, m)	27.80	7.80	43.70	7.06
Basal area (G, m^2/ha)	52.71	16.61	89.90	13.93
Number of trees per ha (N)	1055.28	103.00	7428.00	836.76
Stand volume (V, m^3/ha)	597.97	37.50	1544.00	288.76
$G_{removed}/G_{before\ thinning}$ (%)	7.45	0.00	43.09	6.34
$N_{removed}/N_{before\ thinning}$ (%)	13.06	0.00	78.87	10.58

Table 1.- Summary statistics of the dataset used for model development

To evaluate the presence of autocorrelation and the order of the CAR(x) to be used, graphs representing residuals plotted against lag-residuals from previous observations within each plot were examined visually. The heteroscedasticity was also examined visually, by plotting residuals as a function of predicted values, in order to investigate possible weighting factors. The separated and simultaneous fitting of the equations were accomplished by SAS/ETS® MODEL procedure (SAS Institute Inc., 2009).

Transition functions for dominant height and stand basal area growth

Four different base equations were evaluated for modelling dominant height and stand basal area growth pattern: Korf (1939); Log-logistic; Bertalanffy (Bertalanffy, 1949) and the modified Hossfeld IV function proposed by Cieszewski (2003). These functions have been widely used in modelling stand height and stand basal area growth (e.g., Biging, 1985; Álvarez González et al., 2005; Barrio Anta et al., 2006). Two parameters were considered to be site-specific because equations that possess these characteristics are more flexible, and better describe a wide variety of height- and basal area-age trends.

The formulations of the equations analyzed (E1-E5) are shown in Table 2. Following general notational convention, a_1, a_2, . . . , a_n were used to denote parameters in base models, whereas b_1, b_2, . . ., b_m were used for global parameters in subsequent GADA formulations. All the GADA based models have the general implicit form of

$$Y = f(t, t_0, Y_0, b_1, b_2, \ldots, b_m)$$

Different methodologies have been used to analyze the thinning effects in basal area growth. Some authors (e.g., Knoebel et al., 1986; Barrio Anta et al., 2006) have developed different equations for thinned and unthinned stands. Other studies (e.g., Amateis et al., 1995; Hasenauer et al., 1997) have analyzed the use of a reference growth function to model the basal area growth in unthinned stands and a thinning correction factor to take into account the response of thinning stands. In this work, we used the first approach to take into account the effect of thinning on stand basal area growth, by employing dummy categorical variables. To compare the differences between basal area growth in thinned and unthinned plots, we used the non-linear extra sum of squares method for detecting simultaneous homogeneity among parameters for both treatments (see Bates & Watts, 1988). If the homogeneity of parameters test reveals significant differences between silvicultural treatments, separate basal area growth models are necessary for each treatment.

Transition function for reduction in tree number

In this work, the equation for estimating reduction in tree number was developed on the basis of the following differential functions:

$$\frac{1}{N}\cdot\frac{\Delta N}{\Delta t} = \alpha \cdot N^{\beta} \cdot t^{\delta} \qquad (2)$$

$$\frac{1}{N}\cdot\frac{\Delta N}{\Delta t} = \alpha \cdot N^{\beta} \cdot \left[\gamma + \frac{\delta}{t}\right] \qquad (3)$$

$$\frac{1}{N}\cdot\frac{\Delta N}{\Delta t} = \alpha \cdot N^{\beta} \cdot \delta^{t} \qquad (4)$$

where N is the number of trees per hectare present at age t, $\Delta N / \Delta t$ is the instantaneous mortality rate operating at age t, and α, β, γ and δ the model parameters that regulate the mortality rate.

Equations 2 to 4 consider the mortality rate related to number of trees per hectare and age. Six different transition functions derived from these three differential functions have been used in this study (Table 3).

Static function for stand volume

Estimation of the stand volume is usually required for planning purposes. Three different models, which depend on stand basal area, dominant height, number of stems per hectare and quadratic mean diameter were initially considered for comparison. The formulations of these models are the following:

$$V = a \cdot N \cdot dg^{b} \cdot H^{(3-b)} \qquad (5)$$

$$V = a \cdot G \cdot H \qquad (6)$$

$$V = a \cdot G^{b} \cdot H^{c} \qquad (7)$$

where V is the total stand volume (m^3/ha), G is the stand basal area (m^2/ha), H is the dominant height (m), N is the number of stems per hectare and dg is the quadratic mean diameter (cm).

Model comparison and evaluation

The comparison of the estimates for the different equations fitted for each stand variable was based on numerical and graphical analyses of the residuals (e_{ij}). Three statistical criteria obtained from them were examined: bias ($\overline{E}$), which tests the systematic deviation of the model from the observations; root mean square error (*RMSE*), which analyses the accuracy of the estimates; and the model efficiency (*MEF*), which shows the proportion of the total variance that is explained by the model, adjusted for the number of model parameters and the number of observations. The expressions of these statistics are as follows:

$$\overline{E} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{n} \quad (8)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}} \quad (9)$$

$$MEF = 1 - \frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-p)\sum_{i=1}^{n}(y_i - \overline{y}_i)^2} \quad (10)$$

where y_i, $\hat{y}_i$ and $\overline{y}_i$ are the observed, predicted and average values of the dependent variable, respectively; n is the total number of observations used to fit the function; and p is the number of model parameters.

Once the complete model has been constructed, assessment of their validity is often needed to ensure that the predictions represent the most likely outcome in the real world. The only method that can be regarded as "true" validation involves the use of a new independent data set (Vanclay & Skovsgaard, 1997) but the scarcity of such data forces the use of alternative approaches. Therefore, we decided not to attempt a model validation in this study, and relied on the fact that a well-developed model will behave well. An analysis was also carried out to obtain some information about the predictive ability of the overall stand model. For this, observed state variables from any one inventory of the plots were projected forward for different time intervals using the stand height and stand basal area transition functions. All possible non-descending growth intervals were therefore considered in this analysis. In addition, the values obtained were used to estimate total stand volume at different projection intervals. It must be taken into account that total stand volume is the critical output variable of the whole model, since its estimation involves all the functions included and is important in economic assessments. Bias and root mean squared error of these stand volume estimations were calculated for different projection intervals.

To evaluate the maximum time interval for accurate projections, we used the critical error ($E_{crit.}$). In this study, the critical error is expressed as a percentage of the observed mean stand volume and is computed by re-arranging Freese's χ_n^2 statistic (Robinson & Froese, 2004):

$$E_{crit.} = \frac{\sqrt{\tau^2 \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 / \chi^2_{crit.}}}{\overline{y}} \quad (11)$$

where n is the total number of observations in the data set, *yi* and *ŷi* are the observed and $\overline{y}_i$ the predicted stand volume, respectively, τ is the average of the observed stand volumes, is a standard normal deviate at the specified probability level (τ = 1.96 for α = 0.05), and χ^2_{crit} is obtained for α = 0.05 and n degrees of freedom. If the specified allowable error expressed as a percentage of the observed mean stand volume is within the limit of the critical error, the χ_n^2 test indicates that the model does not give satisfactory predictions; the contrary result indicates that the predictions are acceptable.

The bias, the root mean squared error and the critical error ($E_{crit.}$) for total stand volume predictions were plotted against different time intervals.

Results and Discussion

As a first stage, the transition functions of each state variable (*H*, *G* and *N*) and the static function for stand volume were fitted separately to select the best equation. In a second stage, all that components were fitted simultaneously to minimize the total sum of squared errors in the system.

Initially, only the mean structure of the data was fitted. A trend in residuals as a function of lag-residuals within the same plot was observed in all the models considered for dominant height, basal area, mortality and stand volume. This result was expected because of the longitudinal nature of the data used for modelling each stand variable.

A second-order continuous-time autoregressive error structure (CAR(2)) was required for modelling the inherent autocorrelation of the data of dominant height, stand basal area and total stand volume and a CAR(1) for mortality. After correction for autocorrelation, the trends in the graphs of residuals as a function of age-lag-residuals disappeared. These error structures were also included in the simultaneous fitting.

Transition functions for dominant height and stand basal area

Among all the equations tested for modelling dominant height, model E1 (Table 2), derived from the Korf function resulted in the best compromise between graphical and statistical considerations. Therefore, this model was selected for developing the site quality curves in the simultaneous fitting, and the expression of the transition function finally obtained was the following:

$$Y = \exp\left(X_0\right) \cdot \exp\left(-\left(-433.02 + 1960.43 / X_0\right) \cdot t^{-0.69}\right)$$

$$X_0 = \frac{1}{2} \cdot t_0^{-0.69} \cdot \left(-433.02 + t_0^{0.69} \cdot \ln\left(Y_0\right) + \sqrt{4 \cdot 196043 \cdot t_o^{0.69} + \left(433.02 - t_0^{0.69} \cdot \ln\left(Y_0\right)\right)^2}\right) \tag{12}$$

where Y is the predicted dominant height (meters) at age t (years), and Y_0 and t_0 represent the predictor dominant height (m) and age (years). Equation (12) explained 997% of the total variance of the data, and its RMSE was 0.41 m (Table 4).

Base equation	Parameter related to site	Solution for X with initial values (t_0,Y_0)	Dynamic equation	
Korf: $Y = a_1 \cdot \exp(-a_2 \cdot t^{-a_3})$	$a_1 = \exp(X)$ $a_2 = b_1 + b_2 / X$	$X_0 = \frac{1}{2} \cdot t_0^{-b_3} \cdot \left(b_1 + t_0^{b3} \cdot \ln\left(Y_0\right) + \sqrt{4 \cdot b_2 \cdot t_o^{b_3} + \left(-b_1 - t_0^{b_3} \cdot \ln\left(Y_0\right)\right)^2}\right)$	$Y = \exp\left(X_0\right) \cdot \exp\left(-(b_1 + b_2 / X_0) \cdot t^{-b_3}\right)$	(E1)
Log-logistic: $Y = \frac{a_1}{1 + a_2 \cdot t^{-a_3}}$	$a_1 = b_1 + X$ $a_2 = b_2 / X$	$X_0 = \frac{1}{2} \cdot \left(Y_0 - b_1 + \sqrt{(Y_0 - b_1)^2 + 4 \cdot b_2 \cdot Y_0 \cdot t_o^{-b_3}}\right)$	$Y = \frac{b_1 + X_0}{1 + b_2 / X_0 \cdot t^{-b_3}}$	(E2)
	$a_1 = b_1 + X$ $a_2 = b_2 X$	$X_0 = \frac{Y_0 - b_1}{1 - b_2 \cdot Y_0 \cdot t_o^{-b_3}}$	$Y = \frac{b_1 + X_0}{1 + b_2 \cdot X_0 \cdot t^{-b_3}}$	(E3)
Bertalanffy: $Y = a_2 \cdot (1 - \exp(-a_1 \cdot t))^{a_3}$	$a_2 = \exp(X)$ $a_3 = b_2 + b_3 / X$	$X_0 = \frac{1}{2} \cdot \left((\ln Y_0 - b_2 \cdot L_0) + \sqrt{(\ln Y_0 - b_2 \cdot L_0)^2 - 4 \cdot b_3 \cdot L_0}\right)$ with $L_0 = \ln\left[1 - \exp\left(-b_1 \cdot t_0\right)\right]$	$Y = Y_0 \left[\frac{1 - \exp\left(-b_1 \cdot t\right)}{1 - \exp\left(-b_1 \cdot t_0\right)}\right]^{(b_2 + b_3 / X_0)}$	(E4)
Hossfeld IV: [modified by Cieszewski (2003)] $Y^3 = \frac{a_1 \cdot t^{a_2}}{a_3 + t^{a_2 - 1}}$	$a_1 = b_1 + X$ $a_3 = \frac{1}{2} \cdot b_3 / X$	$X_0 = \frac{1}{2} \cdot \left(Y_0^3 \cdot t_0^{-1} - b_1 + \sqrt{(Y_0^3 \cdot t_0^{-1} - b_1)^2 + 2 \cdot b_3 \cdot Y_0^3 \cdot t_0^{-b_2}}\right)$	$Y = Y_0 \cdot \left(\frac{\left(b_3 + 2 \cdot t_0^{b_2 - 1} \cdot X_0\right) t^{b_2}}{\left(b_3 + 2 \cdot t^{b_2 - 1} \cdot X_0\right) t_0^{b_2}}\right)^{1/3}$	(E5)

Table 2.- Base models and GADA formulations considered for dominant height and stand basal area

Taking into account the values of the goodness-of-fit statistics obtained in the fitting process, the GADA formulation derived from the Bertalanffy base-model (E4 in the Table 2) was selected to estimate the stand basal area growth of Norway spruce plantations in Germany. The expression of the transition function obtained in the simultaneous fitting is the following:

$$Y = Y_0 \left[\frac{1 - \exp\left(-\left(0.014 + 0.001 \cdot I\right) \cdot t\right)}{1 - \exp\left(-\left(0.014 + 0.001 \cdot I\right) \cdot t_0\right)}\right]^{\left(-\left(13.91 - 5.13 \cdot I\right) + \left(80.31 - 24.99 \cdot I\right) / X_0\right)}$$

$$X_0 = \frac{1}{2} \cdot \left(\ln Y_0 + \left(13.91 - 5.13 \cdot I\right) L_0 + \sqrt{\left(\ln Y_0 + \left(13.91 - 5.13 \cdot I\right) L_0\right)^2 - 4 \cdot \left(80.31 - 24.99 \cdot I\right) L_0}\right) \tag{13}$$

$$L_0 = \ln\left[1 - \exp\left(-\left(0.014 + 0.001 \cdot I\right) \cdot t_0\right)\right]$$

where Y_0 and t_0 represent the predictor basal area (m^2/ha) and age (years), Y is the predicted basal area (m^2/ha) at age t, and I is a dummy variable equal to 1 for thinned stands and 0 otherwise

This model explained 99.3% of the total variation in the data and the RMSE was 1.18 m^2/ha (Table 4). The non-linear extra sum of squares method used for detecting simultaneous homogeneity among parameters for thinned and unthinned stands revealed significant differences (the null hypothesis of a unique stand basal area transition function for thinned and unthinned stands was rejected because of a significant F-value of 3.54 at $\alpha = 0.05$).

The results suggest that, for our data set, the stand basal area growth pattern after thinning is slightly higher to the stand basal area growth pattern of a stand with similar stand conditions but that has not been recently treated. This results are in accordance with several studies showing that basal area growth rates in thinned stands exceed those of unthinned stands with the same characteristics (e.g., Amateis et al., 1995; Hasenauer et al., 1997; Pienaar & Shiver, 1984; Pienaar et al., 1985). Based on the database and for a practical use of the whole model, we propose to use the complete equation during the first 3 years after thinning and then equation (13) without the dummy variables for considering the thinning response, although further studies should be developed to analyse the response of this species to different thinning regimes.

Graphical inspection of residual versus predicted values of dominant height and stand basal area indicated that there was no reason to reject the hypotheses of normality, homogeneity of variance and independence of residuals.

Figure 1 shows observed against predicted values of dominant height (left) and stand basal area (right). The linear model fitted for each scatter plot and the coefficient of determination showed good behaviour for both variables. The simultaneous *F*-test for intercept = 0 and slope = 1 showed no bias and provided no reason for rejecting the null hypothesis.

Transition functions for number of stems per hectare

After considering statistical criteria and graphical analysis, model E6 (Table 3) was selected as the equation with the best fit. Therefore, the proposed equation for estimating the reduction of the stem number between two ages (t_1 and t_2) in Norway spruce plantations in Germany obtained in the simultaneous fitting is:

$$N_2 = \left[N_1^{-0.83} + 1.88 \cdot 10^{-7} \cdot \left(t_2^{2.19} - t_1^{2.19}\right)\right]^{-1/0.83} \quad (14)$$

Model	Initial condition	Equation
(E6)	Derived from Eq (2) $b_1 = -\beta \neq 0$ $b_2 = \delta + 1 \neq 0 \quad b_3 = {-\alpha \cdot \beta}/{\delta + 1}$	$N = \left[N_0^{b_1} + b_3 \cdot \left(t^{b_2} - t_0^{b_2}\right)\right]^{\frac{1}{b_1}}$
(E7)	Derived from Eq (2) $\beta = 0.5 \;\; \delta = 1 \quad b_1 = -0.25 \cdot \alpha$	$N = \left[N_0^{-0,5} + b_1 \cdot \left[\left(\frac{t}{100}\right)^2 - \left(\frac{t_0}{100}\right)^2\right]\right]^{-2}$
(E8)	Derived from Eq (2) $\beta = 0 \;\; b_1 = \delta + 1 \neq 0 \quad b_2 = {\alpha}/{\delta + 1}$	$N = N_0 \cdot e^{b_2 \cdot \left(t^{b_1} - t_0^{b_1}\right)}$
(E9)	Derived from Eq (2) $\beta = 0 \quad \delta = 0 \quad b_1 = \alpha$	$N = N_0 \cdot e^{b_1 \cdot (t - t_0)}$
(E10)	Derived from Eq (3) $\beta = 0 \quad b_1 = \alpha \cdot \delta \quad b_2 = \alpha \cdot \gamma$	$N = N_0 \cdot \left(\frac{t}{t_0}\right)^{b_1} \cdot e^{b_2 \cdot (t - t_0)}$
(E11)	Derived from Eq (4) $\beta = 0 \quad b_1 = \delta \quad b_2 = {\alpha}/{Ln\delta}$	$N = N_0 \cdot e^{b_2 \cdot \left(b_1^{t} - b_1^{t_0}\right)}$

Table 3.- Mathematical expression of the mortality models derived from differential equations 2 to 4 used in this study. N and N_0 are the number of trees per hectare present at ages t and t_0, respectively, and b_i are the model parameters

This equation implies that the relative rate of change in the number of trees is proportional to a power function of age. The function explained 99.3% of the total variation in data and the RMSE was 64.0 trees/ha (Table 4). In the whole model, this equation should be used in stands where no thinnings or only light thinnings have been carried out; otherwise, it is better not to consider reduction in tree number, especially after thinning operations.

Examination of the residuals did not indicate any trends in terms of underestimation or overestimation of the number of stems per hectare. Figure 2 (left) shows the observed against predicted values of the number of trees per hectare. The simultaneous *F*-test for intercept = 0 and slope = 1 showed no bias and no evidence for rejecting the null hypothesis.

As previously commented, visual or graphical inspection of the transition functions was considered an essential point in evaluating the overall representation of the real data. Therefore, plots showing the curves for four site indices of 18, 24, 30 and 36 m. of dominant height at a reference age of 80 years, curves of stand basal area increments for thinned and unthinned stands and curves of reduction in tree number curves for 1500, 3000, 4500 and 6000 trees/ha at a reference age of 40 years overlaid on the trajectories of observed values over time, were examined (Figure 3). The curves seem reliable beyond the usual rotation age (100 years), as judged by the estimations of each stand variable overlaid on the trajectories of the observed values over time.

Total stand volume equation

Among the three models analyzed, stand volume equation (7) performed best and was more accurate than the other two models. This model can provide the volume of a stand at a given time once the dominant height and the stand basal area are known for that time. These state variables can be obtained from an ordinary forest inventory in which diameter at breast height and height are measured or can be easily projected over time by their corresponding transition functions. Therefore, the mathematical expression of the stand volume for Norway spruce in Germany obtained in the simultaneous fitting is the following:

$$V = 0.43 \cdot G^{0.97} \cdot H^{1.04} \quad (15)$$

where V is the stand volume (m^3/ha), G is the stand basal area (m^2/ha) and H is the dominant height (m.)

Variable	Model	Param.	Estimate	Approx. SE	Approx. p>|t|	RMSE	MEF	Ē
H	E1	b_1	-433.019	125.421	0.0006	0.4079	0.9970	0.0006
		b_2	1960.431	545.903	0.0003			
		b_3	0.690	0.016	<0.0001			
		ρ_1	0.960	0.008	<0.0001			
		ρ_2	0.917	0.009	<0.0001			
G	E4	b_1	0.014	0.001	<0.0001	1.1824	0.9928	0.0706
		b_2	-13.908	4.849	0.0042			
		b_3	80.312	24.331	0.0010			
		b_{1t}	0.015	0.001	<0.0001			
		b_{2t}	-8.761	1.760	<0.0001			
		b_{3t}	55.323	8.715	<0.0001			
		ρ_1	0.814	0.018	<0.0001			
		ρ_2	0.859	0.018	<0.0001			
N	E6	b_1	-0.825	0.019	<0.0001	63.9750	0.9927	1.1931
		b_2	2.195	0.036	<0.0001			
		b_3	1.875E-7	4.792E-8	<0.0001			
		ρ_1	0.611	0.022	<0.0001			
V	(7)	a	0.431	0.007	<0.0001	8.9416	0.9990	0.2374
		b	0.966	0.006	<0.0001			
		c	1.040	0.005	<0.0001			
		ρ_1	0.999	0.004	<0.0001			
		ρ_2	0.946	0.004	<0.0001			

Table 4.- Parameter estimates and goodness-of-fit statistics for the prediction functions selected. (SE, standard error; RMSE, root mean square error; MEF, model efficiency)

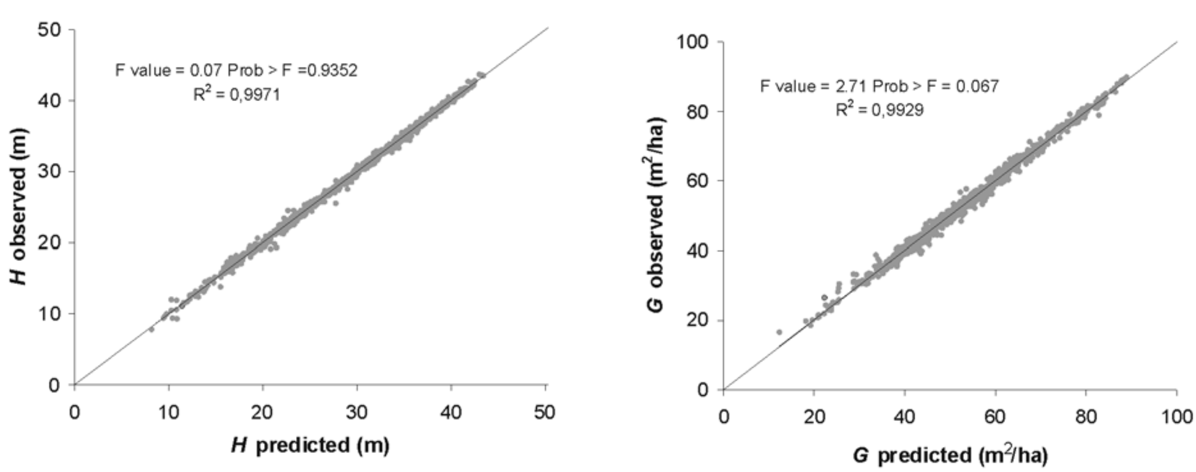

Figure 1.- Plot of observed versus predicted values of dominant height (left) and stand basal area (right). The solid line represents the linear model fitted to the scatter plot of data and the dashed line is the diagonal. F value and its associated probability are the results of the simultaneous F-test for intercept=0 and slope=1

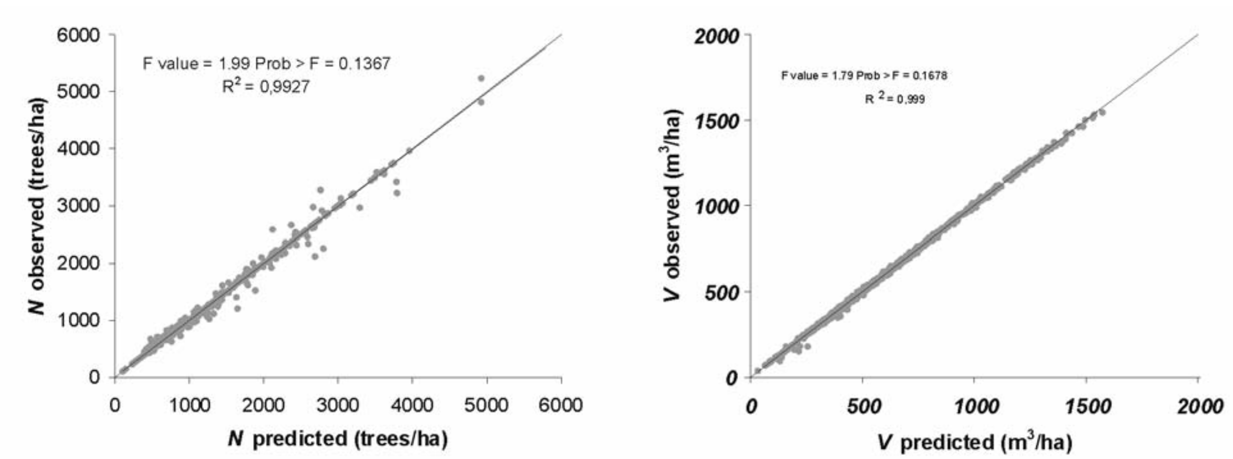

Figure 2.- Plot of observed versus predicted values of number of trees per hectare (left) and volume (right). The solid line represents the linear model fitted to the scatter plot of data and the dashed line is the diagonal. F value and its associated probability are the results of the simultaneous F-test for intercept=0 and slope=1

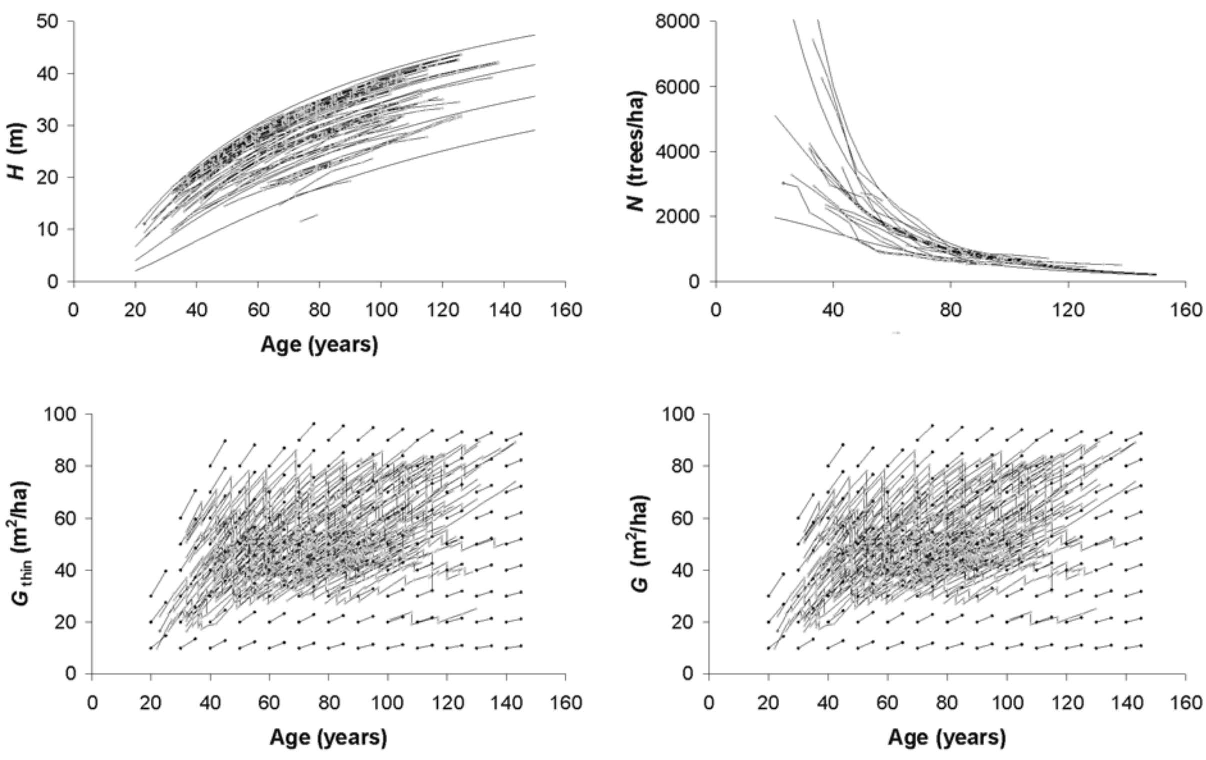

Figure 3.- Upper left: Dominant height curves for site indices 18, 24, 30 and 36 m at a reference age of 80 years overlaid on the trajectories of observed height over time for the GADA equation E1. Upper right: Reduction in tree number curves for 1500, 3000, 4500 and 6000 trees per hectare at a reference age of 40 years for the ADA equation E6. Down: Basal area increments obtained using the GADA equation E4 for thinned (left) and unthinned (stands) overlaid on the trajectories of observed basal area over time

The model provided a very good performance, explaining 99.9% of the total variance with a RMSE value of 8.94 m^3/ha (Table 4). Plots of residual versus predicted stand volume did not show reasons to reject the hypotheses of normality, homogeneity of variance and independence of residuals. Plot of observed versus predicted stand volume for the equation selected is shown in Figure 2 (right). This model also provided a random pattern of residuals around zero with no detectable significant trends for predicted stand volume.

Overall evaluation of the model

To assess if the model satisfies specified accuracy requirements, observed dominant height, number of trees per hectare and stand basal area from any one inventory of the sample plots were projected forward for different time intervals using the corresponding transition functions (Eqs. (12), (13), and (14)). All possible non-descending growth intervals were therefore considered in this analysis. Equation (15) was then used to estimate the total stand volume and equations (8), (9) and (11) were used to calculate bias, root mean square error and critical error in stand volume estimations.

Plots of bias and root mean squared errors in the stand volume estimates for different projection intervals showed that there was no systematic over- or underestimates of stand volume for prediction intervals up to 20-25 years; however, there was a slight tendency towards underestimation for projection intervals increasing (values obtained for projection intervals longer than 70 years are not considered because the reduced number of observations).

In forestry growth modelling, the required accuracy in which a mean prediction error of the observed mean at 95% confidence intervals within $\pm$10-20% is generally realistic and reasonable as a limit for the actual choice of the acceptance and rejection levels (Huang et al., 2003). As can be observed in Figure 4, stand volume estimates should not be made directly for differences in ages of more than 25 years for not exceed a critical error (E_{crit}) value of 15%. For longer projection intervals, new inventory data should be used to establish a new starting point for estimations.

In conclusion, the proposed model may provide a satisfactory basis for embedding into landscape-level planning models and other decision support systems that enable forest managers for comparing management alternatives in Spruce stands.

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