INVARIANCE, SYMMETRY, AND LAWFULNESS

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Resumen

El propósito principal de este artículo es precisar la tesis, adelantada por Woodward (1992), de que la noción clave para caracterizar a las leyes físicas es la de invariancia. Para elaborar lo anterior, se distinguen dos niveles de invariancia: primero, que las leyes físicas valen invariablemente en cierto dominio de aplicación y, segundo, que ellas cumplen con algunos principios de simetría. Se sostiene que esos dos niveles de invariancia son sellos distintivos del estatus nómico de las leyes básicas de las teorías físicas. La concepción aquí adoptada y elaborada se contrapone con la tesis filosófica tradicional que mantiene que las leyes científicas son enunciados universales verdaderos con carácter necesario, que las distingue de las generalizaciones accidentales verdaderas, las cuales son contingentes.
Palabras clave: leyes físicas, transformación simétrica, validez invariante.

Abstract

In this paper I attempt mainly to elaborate the thesis, advanced by Woodward (1992), that the key notion to depict physical laws is that of invariance. I draw a distinction between two levels of invariance in order to elaborate that thesis. I maintain that distinctive marks of the nomic status of basic laws of physics are either that they hold invariantly, within a domain of application, or that they fulfill some principles of symmetry. The former mark relates to the manner in which physical systems change invariantly whereas the latter concerns to the invariance of the laws themselves. This view contrasts with the traditional philosophical thesis that scientific laws are true universal statements with a necessary character which differentiate them from accidental true generalizations which are contingent.
Keywords: physical laws, symmetrical transformation, invariance holding.

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Introduction

Since several decades the question about the symmetry of physical laws has become a central topic of present physics (see, e. g., Feynman 1963, p. 52-1). The relevance of this physical topic to the philosophy of science resides in that it offers an alternative view to a current philosophical view on the nomic status of such laws. According to that current view, laws are necessarily true universal statements: The peculiar feature of laws which distinguish them from accidental true generalizations —general statements which express contingent facts as "all bodies consisting of pure gold have a mass less than 100,000 kilograms", which could be true as a matter of fact (see Hempel 1966, sec. 5.3)— consists in the necessary character of the universal statements of science.[1] There is a variety of versions of that view about the laws. Though with substantial differences, the following philosophers share, among many others, the claim that laws are or have to be necessary in character: Armstrong 1983; Dretske 1977; Swoyer 1982; Shoemaker 1998 and Bird 2005. On the alternative view we can say that the laws of physics are universal statements that hold invariantly. In contrast to that received view, which comes from an old philosophical tradition —a scientific realist viewpoint—, the invariance view can be extracted from the actual physics, and because of this it turns out to be more appropriate and in agreement to present physics. In this paper we explore that view about laws of physics and, at once, we discuss some relevant and valuable theses due to James Woodward who poses the issue of the relationship of invariance with lawfulness. I share with him the claim that: "[…] the key notion for understanding laws is the notion of invariance rather than the metaphysical notion of a necessary connection (conceived as something distinct from invariance)." (1992, pp. 210-211). In order to have some background for the previous, it is convenient to begin with an explication of the related notion of symmetry and with a very briefly exposition of some important symmetries in physics. Our main aim here is just to elucidate slightly the relations among the concepts of nomicity or lawfulness, symmetry and invariance.

[1] It is not of our concern here other conceptions of laws such as the regularity view in Humean vein. For a sound criticism of various conceptions of laws, see van Fraassen (1989).

Symmetry

Herman Weyl characterizes a classical concept of symmetry as follow: "A thing is symmetrical if one can subject it to a certain operation and it appears exactly the same after the operation."[2] A more precise concept is the following: "The definition of symmetry as 'invariance under a specified group of transformations' allowed the concept to be applied much more widely not only to spatial figures but also to abstract objects such as mathematical expressions —in particular expressions of physical relevance such as dynamical equations." (Brading et al 2017, pp. 2-3). The relevant mathematical concept of group is defined by the three following conditions: a family of functions G of a set over itself is a *group* if (1) G contains the identity transformation I, i.e., for all x, $I(x) = x$. (2) G is closed with respect to product: if f and g are in G, then there is another transformation gf such that for all x, $gf(x) = g(f(x))$. (3) G has inverses: if f is in G, then there is another transformation f^{-1} such that $ff^{-1} = f^{-1}f = I$ (van Fraassen 1989, p. 245).

It is worth to note that the invariance alluded allows to understand it both in a physical and in a conceptual sense, whereas the transformations involved do not mean necessarily some physical change but only, in some cases, a correspondence. 'Transformation' could be substituted by 'operation' or 'function' in their mathematical senses (even in some cases the operations could be performed with paper and pencil). We would use here the terms transformation and operation interchangeably. Some physicists use the terms 'symmetry' and 'invariance' as synonymous, but here I will differentiate them following Pierre Curie´s theses: "The symmetries are in the laws of the phenomena, not in the phenomena themselves" and "The phenomena breaks the symmetries of laws"[3] –thus, we will consider the symmetries only with respect to the laws whereas the invariance in relation either to the physical systems or to the laws which hold on these systems.

In order to delimit the scope of our discussion on the symmetry of laws, from here on we shall refer to laws, principles and equations of physical theories which Roger Penrose calls 'superb', those which he characterized in terms of an extraordinary wide range and accuracy of application. This category of theories included Newton mechanics, electromagnetism, special

[2] Quoted by Feynman et al (1963, pp. 11-1) from Hermann Weyl. *Symmetry*. Princeton NJ: Princeton University Press, 1952.

[3] Quoted by Earman (2004, p. 1231) from Pierre Curie, "Sur la symétrie dans les phénomenes physiques". *Journal of Physique* 3. 1894, pp. 393-417.

and general theories of relativity, as well as quantum mechanics and electrodynamics (Penrose1989, p. 152).[4]

Symmetry in Physics

Richard Feynman points out several symmetrical transformations as examples of operations under which diverse physical phenomena are invariants, which are the following: translation in space, translation in time, rotation in space, uniform velocity in straight line (Lorentz transformation), reversal of time, reflection in space, interchange of identical atoms or identical particles, quantum-mechanical phase, and matter–antimatter (charge conjugation) (1963, p. 52-2).

The first symmetry designs a translation of a physical system in space, by displacing the spatial coordinates. In such case Feynman remarks that if the Newton´s laws hold in a set of coordinates, they also hold in any other set of coordinates (1963, p. 11-6). Or, equivalently: the holding of Newton´s laws are *independent* from the set of coordinates that one selected. One can make similar claims with respect to translation in time —which consists in changing the temporal origin— and rotation in space —which consists in modifying the orientation of the coordinate system. These cases of symmetries are significant instances of the general thesis: If a class of laws hold on a sort of physical processes or phenomena, they hold *invariantly* under a specific group of transformations of those processes or phenomena.

We could think of performing the three former symmetries in any order in a classical system. By the condition (2) of the prior definition of group, the product of the three corresponding operations yields a system which is equivalent to the original in the sense that the same laws hold on both. This is a consequence of the claim that the holding of classical laws is independent of the spatial and temporal coordinates. It is worth to note that, though we do not perform such transformations, their realization has full physical sense. This contrast with another triad CPT of symmetrical transformations —charge conjugation C, reflection of space P, and reversal of time T— in quantum systems, whose operations are only paper and pencil operations, as we shall see below.

[4] Penrose excluded thermodynamics because: "In my view, thermodynamics, as it is normally understood, being something that applies only to *averages*, and not to the individual constituents of a system […] is not quite a physical theory in the sense that I mean here (the same applies to the underlying mathematical framework of *statistical mechanics*)" (1989, p. 221, fn. 2).

The former examples are only simple cases of symmetries which Feynman and coauthors consider of geometrical nature (1963, p. 52-3). The classical or Galilean principle of relativity provides physical content to these symmetries. The Galilean transformations between classical systems of coordinates are symmetrical transformations, and form a group under which distance, relativity velocity, and acceleration are invariant (see van Fraassen 1989, p. 272). The principle of relativity of Einstein´s special theory asserts that the laws that prescribe changes in physical systems are independent of whether they refer to one or other coordinate systems which travel the one with respect to the other with a uniform translation movement. Together with the postulate about the constancy of light velocity, this principle involves Lorentz transformations, i. e., transformations between four-dimensional coordinate systems which move uniformly the one to respect to the other. According to special theory of relativity, the physical laws are invariant under such Lorentz transformations (in a Minkowski spacetime), which form a group (see Brading et al 2017, p. 5) When the velocity is small compared with light velocity, a Lorentz transformation reduces to a Galilean transformation.

Certain quantum equations about the gravitational, electromagnetic, strong and weak interactions in elementary particles systems are invariant under symmetrical transformations. From the previous list of symmetry operations, let us pick out the following: matter-antimatter exchange (charge conjugation C), reflection of space P, and reversal of time T. The transformation C consists in an exchange of particles by their antiparticles; P, also named ‘parity’, is a transformation of the coordinate system replacing (x, y, z) by ($-x$, $-y$, $-z$), whereas the operation T is performed substituting the time parameter t by $-t$. Some of them deserve special mention because they have exceptions:

> A physical interaction that is invariant under the replacement of particles by their antiparticles (and vice-versa) is called C-invariant. This operation of spatial reflection (reflection on a mirror) is referred to as P (which stand for *parity*). […] ordinary weak interactions are not invariant under either P or C separately, but it turns out that they are invariant under the combined operation CP (=PC). (Penrose 2004, p. 638)

The more known asymmetry is about spatial reflection P when the parity in certain weak interactions, as in the beta decay, is not preserved. This is the right-left asymmetry, where the parity P is violated. Also, the symmetry of reversal of time transformation T deserves special consideration. According to Feynman and coauthors, in contrast to what happens in classical world, quantum equations hold under the symmetrical transformation of reversal of time. That is, though it is physically impossible to invert the direction of time, at a theoretical level these equations are reversible with respect to time. They comment that:

> Next we mention a very interesting symmetry which is obviously false, i. e., *time reversal*. The physical laws apparently cannot be reversible in time, because, as we know, all obvious phenomena are irreversible on a large scale. […] But if we look at the individual atoms themselves, the laws look completely reversible. This is, of course, a much harder discovery to have made, but apparently it is true that the fundamental physical laws, on a microscopic and fundamental level, are completely reversible in time! (Feynman et al 1963, pp. 52-4)

Let us now consider the product of the following transformations: replacement of particles by their antiparticles C, reflection of space P and reversal of time T. Let us denote by –**S** the system which results from performing these three symmetry operations on a physical system **S**. The quantum equations that hold on a physical system **S** hold invariably on –**S**, and vice-versa. In a theoretical sense, the invariance of some fundamental laws is preserved under the product of the parities C, P and T. The theorem CPT of quantum field theory establishes that: it asserts that every physical interaction is invariant under the product of these three symmetry operations (see Penrose 2004, 638).

A conclusion can be drawn from the previous material of physics: When some laws hold on certain kinds of processes, they hold invariantly under specific groups of transformations of those processes.

Invariance and Woodward´s theses

One of Woodward´s central theses is that: "[I]n actual scientific practice, the notion of lawfulness is closely connected with the notion of stability and invariance." (1992, p. 202).[5] I think that this claim is right, though it can be improved if we distinguish two levels of connection between lawfulness and invariance, as seems that Woodward implicitly does. With respect to the first level he says:

> The idea of a law as an invariant relationship is closely tied to the distinction between laws and initial conditions. We think of the initial conditions describing a physical

[5] Indeed, Woodward's main thesis concerns to the ground of laws on facts about the capacities and powers of particular systems. It is not of our interest here to discuss this claim on the nature of laws. Anyway, it seems that he does not insist on this thesis in his later works, e. g. (2003), with the exception of (1993).

> system as capable of freely varying or as capable of assuming any one of a range of different values, but the law itself does not undergo similar variations. (1992, p. 203)

It is feasible to consider that the laws that fulfill the prior idea are the dynamic laws, laws which prescribe the evolution or transition of physical systems from a given initial state to a final state, such as Newton´s laws and Schrödinger´s equation. The relevant notion of invariance in such cases is that these laws hold in all instances of approximate application to a physical system within the intended domain of application of the pertinent theory: the invariance consists then in the holding of the laws constrained to the admissible ranges of values of the magnitudes involved. As Woodward points out: "[…] invariance is always invariance with respect to some specific class of changes or interventions, which we can think of as constituting the domain of applicability (or "regime") of the law." (1992, p. 203), but beyond these ranges the invariance breaks down, as in the case of the ideal gas law at high densities on which intermolecular forces are relevant (1992, p. 203). Likewise, if the velocity is close to the speed of light the invariance of the holding of Newton´s laws and of Galilean principle of relativity break down.

From a methodological point of view, this order of ideas suggests, to me, that in practice the physicists seek to know the scope of the holding of the laws within the intended domain of application of the pertinent theory. The actual scope of application of a set of laws will be delimited by the kinds of physical systems where the holding of these laws is invariant, i. e., where the laws do not break down.

At a second level one finds the sort of invariance associated with the principles of symmetry, as expressed by the cited definition of symmetry as invariance under a specific group of transformations. Woodward indicates this second level when he says that:

> One of the features of fundamental physical laws most emphasized by physicist themselves is that laws must satisfy various symmetry requirements. Such symmetry requirements are in effect invariance requirements: they amount to the demand that laws express relationships that remain invariant under certain kinds of transformations or changes. These transformations are usually specified group-theoretically. (1992, pp. 203-204)

The two levels of invariance are indeed different. The first one refers to an invariance on the manner in which physical systems change in accordance with some law. The second one refers to a higher invariance, to an invariance of the laws themselves, associated with the symmetry operations such as translation in space, translation in time, rotation in a fixed angle, inversion of time, reflection of space, and exchange of matter-antimatter (charge conjugation). Thus, on the one hand, the *relata* of the invariance are the physical systems and the processes that they suffer, which is expressed by a law of evolution which prescribes the possible changes of state of the systems allowable by such law. One the other hand, at a high level of abstraction, the *relata* of the invariance are the laws themselves that are expressed by principles of symmetry which state the sort of transformations under which the holding of the laws of evolution is preserved.

As I said, Woodward refers to those two levels although in a tacit manner without separating them clearly, as the following quote indicate:

> To say, for example, that the ideal gas law (PV = *n*RT) is a law is to say not just that it describes […] the relationship between pressure volume and temperature which obtains in various actual samples of gas, but that the relationship will remain approximately the same if various changes were to occur —that the relationship would continue to hold if we were to move some sample S from its present location to a new location, or if we were to double the pressure in sample S or reduce its temperature. (1992, p. 202)

We can see there the two senses of the notion of invariance: the change of location of S designs a symmetry operation —translation in space— whereas to reduce the temperature of S is a change on the initial conditions related with the invariance of the holding of that law.

Both levels of invariance are related to principles of conservation of magnitudes as momentum, energy, and charge. In a first instance, there is a local sense in which the conservation principles assert that the total amount of the magnitude under consideration is preserved in all the physical systems which suffer processes according to a law of evolution. There is, besides, a global sense on which the conservation principles are linked with the principles of symmetry, as Beiser says: “Every symmetry operation corresponds to something being conserved, though not necessarily in every interaction” [6] (1987, p. 537), or on van Fraassen´s slogan: “For every symmetry a conservation law.” (1989, p. 287). At this abstract level, the invariance of the laws under a specific group of transformations, expressed by the principles of symmetry, is connected with the preservation of the amount of such magnitudes. Some magnitudes are conserved under some specific symmetries; among others, the linear momentum is conserved under translation in space, the energy is conserved under translation in time, the angular momentum is conserved under rotation in space, electric charge

[6] However, perhaps the converse is not sustainable, because, e. g., the baryon number B is conserved but it is unknown to what symmetry it could be associated (1987, p. 537).

is conserved under electromagnetic gauge transformation, and the product of charge parity, space parity, and time parity are conserved under inversion of charge, space, and time (see Beiser 1987, p. 538).

The difference between both levels of symmetry can be deemed with respect to the types of questions that could be relevant to answer. On the one hand, questions about how a physical system, of certain sort, changes from an initial state to a final state, or about why a process, of certain sort, occurs in some physical systems could be responded by appealing to the invariance expressed by laws of evolution. On the other hand, questions about how one can generalize to the whole universe, in space and time, the discoveries about natural phenomena observed at a local place, or how one can infer a general knowledge from experiments and observations performed here and now, could perhaps be answered calling out the principles of symmetry which express the invariance of translation in time and space, and the invariance of the exchange of particles by identical particles and other kinds of invariance as well.

Thus, we find some important links between the concept of lawfulness and the concepts of invariance and symmetry. Distinctive marks of the nomic status of physical laws are the invariance of their holding in the pertinent physical systems as well as their invariance under symmetrical transformations. We could say that a peculiar feature of the *lawful status* of the basic equations of physical theories, as those called 'superb' by Penrose, resides in their invariance under transformations of symmetry. However, as it is known, this claim has exceptions such as the violation of the parity P in weak interaction of radioactive decay —as Beiser says: not all symmetry operation holds in every interaction.

From all the previous —the invariance of the laws of evolution, the principles of symmetry, and the principles of conservations together—, it must be patent that accidental generalizations, even true, do not fulfill the two former marks, and therefore we can distinguish the laws of physics from them without appeal to any kind of necessity.

Idealized laws

There is another important issue which deserves our attention since it becomes relevant to the character of physical laws. It is about the abstract and idealized character of fundamental laws of physics:

> Consider another typical example of a fundamental law of nature: Schrödinger´s equation. This law is, in a variety of ways, extremely general and abstract. If we

> wish to use this law to analyze some specific system [...] we must make a number of additional, much more specific assumptions to insure the applicability of the law. (Woodward 1992, p. 195)

The main task to make in order to approximately apply Schrödinger´s equation to a physical system consists in choosing a specific Hamiltonian for the system, since this equation is like a *general schema* which has to be filled in or specified in a variety of ways so as to yield descriptions of real systems (1992, p. 195). Further, in order to get some statements with a specific and concrete physical content from the Schrödinger´s equation, Woodward points out: "we must also make a number of other specific assumptions, usually involving large elements of simplifications and idealizations, about initial and boundary conditions." (1992, p. 195).

The work of Leszek Nowak (1992; 2000) becomes relevant to this topic about the idealized laws, and how one could apply them to concrete physical systems. He has proposed a procedure in order to obtain statements with factual content from abstract and idealized statements: a method of concretization. Let me expose briefly Nowak´s method.[7]

Nowak draws a distinction between two concepts: "Roughly, abstraction consists in a passage from properties *AB* to *A*, idealization consists in a passage from *AB* to *A*–*B*" (2000, p. 8). In both cases, the property *A* has been selected but in the passage of abstraction the property *B* is simply *subtracted*, while in the passage of idealization the property *B* is *negated* or, more likely distorted. According with him, the properties subtracted or negated can generate counter-actual statements by means of the introduction of idealizing conditions of the form $p(x) = 0$. This procedure can render law-like statements which Nowak calls idealizational: "An idealizational statement is a conditional possessing an idealizing condition in the antecedent." (1992, p. 11). The procedure of concretization consists of removing such condition and replacing it with its realistic negation, and introducing a correction in the formulae (consequent) of the statement (see Nowak 1992). In this manner, the concretization procedure leads to a more realistic statement than the initial idealizational statement. The general form of laws is that of an idealizational statement: If ($G(x)$ & $p_1(x) = 0$ & $p_2(x) = 0$ &...& $p_{k-1} = 0$ & $p_k(x) = 0$ then $F(x) = f_k(H_1(x), \ldots, H_n(x))$, where $G(x)$ stand for a realistic and actual condition, each H for a factor which is considered as principal, each $p_i(x) = 0$ is an idealizing condition, and therefore the whole law-like statement asserts a functional relationship f between the

[7] For a study of Nowak's idealizational approach to science, see Borbone (2011).

magnitude F and the principal factors H, which is expressed by the formulae occurring in the consequent (see Nowak 2000, pp. 9-10).

Basically, Nowak thought of a sequence of statements which leads to a final factual statement via the procedure of concretization, that is to say by means of elimination, step by step, of the idealizing conditions in the antecedent, restoring its realistic versions, and consequently, introducing a modification in the functional formulae in the consequent, until obtain a statement which, according with him, lacks of any idealizing conditions and is a factual statement (see 2000, pp. 9-10).

Nowak introduces a further element, an approximation procedure: "Normally, however, final concretization is not met in science. Normally, after introducing some concretizations the procedure of approximation is being applied. That is, all idealizing conditions are removed at once and their joint influence is assessed as responsible for the deviations up to certain threshold ε." (1992, p. 12).[8] The result is still an idealizational statement which increases, in some degree, the applicability of the theory to the facts.

I point out elsewhere (Rolleri 2013), that Nowak in his notion of idealizing condition confuses subtracted and distorted properties. That is, in the formulation of the antecedent of an idealizational law, Nowak does not distinguish between an idealizing condition derived from a neo-Leibnizian idealization (a distortion, a deliberate falsification) and an idealizing condition resulting from a neo-Hegelian abstraction (which select some property separating the essence from the appearance). In other words, he fails to differentiate counterfactual suppositions, distinctive of idealization, and counter-actual suppositions, peculiar of abstraction. As a consequence, in his analysis of the concretization procedure, Nowak mingles a combination of de-idealizing steps and concretization steps. In the following example, he considers the frictionless supposition as an idealizing condition and the inverse procedure a case of concretization: suppose that the system is a classical body in free fall, then the concretization step would consist in move from the statement 'if $ff(x)$ & $R(x) = 0$ then $s(x) = 1/2\ gt^2$' to the statement 'if $ff(\mathrm{x})$ & $R(x) = r$ then $s(x) = 1/2\ gt^2 — r$', where R stand for the medium resistance (2000, 8). In this case the resistance medium is just subtracted, and then $R(x) = 0$ is rather an abstraction condition.

[8] Moulines objects that this notion of approximation reduces the role of approximation to a secondary role of "epsilontics" in the process of matching theory with experience (2007, p. 258).

It seems that not all idealizing conditions can be eliminated by Nowak´s procedure of concretization but just those that correspond to missing factors which have been subtracted. For example, the idealized suppositions in classical mechanics of material bodies as masses concentrated at extensionless points, rigid bodies, and perfectly elastic spheres cannot be removed by simple steps of concretization. An example more illustrative consists in Bohr´s unintentional supposition in his original model of hydrogen atom that the electron has a definite classical trajectory in the stationary states (see McMullin, 1985). Nowadays, this supposition turns out to be 'unrealistic' because the description that quantum mechanics gives to these trajectories is in terms of cloud of probability density around the nucleus. This case shows that some idealization of the system which involves a distortion (an idealizing condition, in Nowak's terms) cannot be eliminated by the concretization steps but rather a reconceptualization of the system in question is required, under different suppositions, which would involve a novel idealization of the system. Thus, Nowak's method of concretization applies well on factors which has been subtracted but not so, at least in some important cases, on factors which has been idealized.

The former suggests that Nowak´s thesis that by the method of concretization we can obtain factual statements exempt from any unrealistic supposition is a simplification, a legitimate although idealized thesis itself. His proposal about the approximation procedure makes such method more plausible because allows us not to eliminate all missing factors, to remove all idealized suppositions from the fundamental laws of physics.

Nancy Cartwright maintains a similar assertion to Woodward's with respect to Schrödinger´s equation. She says that the theoretical laws of physics do not apply directly to physical processes, neither through deductive arguments, nor by any other means and so rejects that theoretical laws could be true in a factual sense, true with respect to the facts (1983). Moreover, reaffirming the thesis that "The basic scientific laws do not literally describe the behaviour of real systems. They are instead to be taken as abstract claims." (1989, p. 203), Cartwright pursued the problem involved by following Nowak's method of concretization. In her approach, the concretization procedure begins with the specification of the Hamiltonian *H*, which stands for the total energy of the system, from an available list provided by the theory: Hamiltonians for the hydrogen atom, for the square-well potential, for the harmonic oscillator, and so forth (1989, p. 205). Thus, her contribution consists in a proposal of how that method can be carried on in quantum mechanics. Schrödinger´s equation $H\phi = —ih\vartheta\phi/\vartheta t$, the fundamental law of quantum mechanics, is used to obtain models such as the basic ones: the central potential scattering, the Coulomb interaction, the harmonic oscillator and the kinetic energy. Once that a specific Hamiltonian has been introduced one can apply Nowak's method in order to eliminate some abstracted factors. For example, for a single particle with mass *m* moving in an electric field whose contribution to the energy is $V = V(x, y, z)$, the factor H' to be add is $V\phi$ to the principal Hamiltonian H_0 equal to $-\hbar^2/2m\nabla^2\phi$, and so Schrödinger´s equation adopts the form of $H\phi = -\hbar^2/2m\nabla^2\phi + V\phi$. In general, in Schrödinger´s equation the Hamiltonian H is equal to $H_0 + H'$, where the H's are meant for the correction additions in concretization procedures (see Cartwright 1989, p. 205).

However, for Cartwright, what we can get via concretization procedures, are laws applicable only to abstract models not to real object. No mind how far we go in the stages of concretization, we will never go far enough to be able to describe concrete singular systems: "[…] no matter how open-ended the list is, this kind of process will never result in an even approximately correct description of any concrete thing. For the end-point of a theory-licensed concretization is always a law true just in a model" (Cartwright 1989, p. 207).

Addition support in favor of the thesis that fundamental laws of physics are general schemes of statements can be found in the position that Kuhn advances when he says that basic physical laws, the "symbolic generalizations", are rather law-sketches or law-schemas instead of law statements. Newton´s second law, in particular, adopts different forms according to the specification of the sort of forces involved. For example, $f = ma$ becomes $mg = d^2s/dt^2$ for a body in free fall, and $mg \sin\theta = -ml\, d^2\theta/dt^2$ for a simple pendulum, and so on (cf. 1970, p. 188).

Following Kuhn´s hints with respect to the symbolic generalizations of physics, Moulines offers an analysis of both the function and logical form of Newton´s second law and maintains that the work of such law consists in being a guiding principle for further applications and, eventually, novel applications. He analyzed the logical form of Newton´s concept of force which corresponds to a function of tuples of functions, named 'functional', and not to a function of individual variables, whose arguments are values of other parameters —parameters like spatial coordinates, instants, velocities, masses, electric charges, magnetics poles, and so on— which are functions of particles and times (see Moulines 1982, chaps. 2.3 and 2.5; also Moulines 1984). He claims that for the high degree of abstraction of Newton´s concept of force, the second law is a general schema which is empirically unrestricted and immune to refutation. The reason of the former resides in that a complete mathematical formulation of Newton´s second law comprises both a number of existential quantifiers, one for each parameter, and a second order existential quantifier for the functional 'force'. Hence, Moulines argues that the introduction of a number of first order existential quantifiers reduces substantially the empirical content and, at the same time, the inclusion of an existential quantifier for the functional force would turn the search of counterexamples which would refute such law into an absurd, because as the quantifiers involved are existential, for mathematical reasons, the possibilities of choice either functions for parameters or functional for forces are infinitely super-denumerable (cf. 1982, p. 106).

If that is so, then when the physicists recurred to Newton´s second law in order to apply it, together with some special laws, to a novel kind of phenomena, they were not testing that fundamental law but rather they were trying to find out whether the bunch of those laws hold on that kind of phenomena, trying to delimit the actual scope of those laws —not to refute them as Popper maintained. Of course, this leave place to reply that some special laws involved, supplementary to the fundamental law, are indeed being tested in the sense of an attempt to find out its truth value. Nevertheless, this objection disregards the approximate character of every claim which can be derived or obtained from the fundamental laws of physics by means of procedures as concretization, specialization or approximation[9] —character which is not in accord with the realistic view of truth as correspondence to world facts. What one may expect is that singular physical claims be in agreement with empirical data.

Moulines does not pose there the issue about the truth values of law-like statements which can be obtained from the Newton's second law by such procedures of specialization and approximation. Moreover, the structuralist philosophers of science are not concerned with the question about the true of physical laws (cf. Balzer, Moulines and Sneed, 1987) nor is Kuhn. Recall Kuhn´s assertion that the very idea of a match between the ontol-

[9] The procedure of specialization consists in adding some special laws to the basic ones in order to derive law like statements with more specific content, whereas the procedure of application approximation consists in the application of laws to a given systematized collection of empirical data which once has been obtained by the procedure of model construction approximation which consists in systematizing "some empirical data within a given conceptual framework and in the process we have to make some 'idealization' and 'simplifications´ (read: approximation) in order to obtain a manageable model" (Balzer, Moulines and Sneed 1987, pp. 170 and 325).

ogy that a theory postulates and what is “really there” is only an illusion (cf. 1970, p. 206).

As I agree with all the previous order of ideas, I conclude that the basic nomic equations of theoretical physics are not indeed statements but schemas of statements instead, which do not admit truth values.[10]

Lawfulness and counterfactuals

So far, I have tried to elucidate a bit the concept of invariance, in connection to the lawful status of physical equations, making explicit two important senses or levels of invariance which are related with two kinds of laws or principles: laws of evolution and principles of symmetry. Now I wish to expose some remarks to Woodward´s thesis about the link between laws or invariant generalizations and counterfactual statements.[11]

Among several criteria that philosophers have proposed in order to differentiate scientific laws from accidental true generalizations such as that the laws are exceptionless generalizations expressed by universally quantified conditionals, that they make no reference to particular objects or spatiotemporal locations, that they have very wide scope, that they support counterfactuals, that they are confirmable by their instances, and that they are integrated into a body of systematic theory, Woodward finds relevant only the criterion that laws support counterfactual statements, where the relevance involved consists in whether a generalization counts as invariant (2003, pp. 265-66) —under the thesis that: “Laws may be distinguished from accidental true generalizations by their invariance properties.” (1992, p 205). Moreover, Woodward writes:

> I should make it explicit that the notion of invariance and stability are counterfactual notions: what matters is not just whether the relationship expressed by a law holds under the range of circumstances that actually occur. It also matters whether that relationship would continue to hold under various physically possible circumstances and changes different from those that actually obtain. (1992, p. 202)

[10] Of course, this is contrary to the realistic thesis that laws are *true* universal statements. However, the contemporary debate about scientific laws between realistic and antirealistic philosophers is beyond of our purposes here.

[11] Woodward (2003) has elaborated an interventionist view of explanation and causation which intent to be applicable to the special sciences as economy and genetics. I have nothing to say here about the laws or principles of such sciences which he considers as explanatory because they are invariant generalizations.

Thus, he understands the notion of counterfactual with respect to that of invariance in an extended sense which differs from the traditional notion; he explains that: "Again, what matters is not whether we or nature will ever actually produce such changes or possible circumstances (or whether it is technologically feasible for us to produce these), but rather what would happen if (never mind how) such changes or circumstances were to occur." (1992, p. 202).

Woodward presents the traditional view about the connection of laws and counterfactuals as follow. Laws are, first at all, universal statements of the form (1) All *As* are *Bs* (i. e., for all *x*, if *Ax* then *Bx*) that could support counterfactual statements of the form (2) "If this *x* were to be an *A*, then *x* would be a *B*, where *x* is any arbitrary object, including (especially) those that are not at present *As*." (2003, p. 279). In contrast to this view, Woodward proposes the view that attributes a special significance to counterfactuals that describe what would happen to *B* under *interventions* that bring about *A*. (2003, p. 280).

As an illustration, let us consider the following formulation of the law of thermal expansion: For all *x*, if *x* is a metal bar with length L_0 and if the temperature of *x* raises from T_0 to *T* then the length of *x* changes by $L = kL_0T$, where *k* is a constant which expresses the thermal expansion coefficient of the metal *x*. Under the traditional view this law supports counterfactuals of the form: If *y* were a metal bar … and … then the length of *y* would change by $L = kL_0T$. Now, if an object *y* is a metal bar, then the consequent of a statement obtainable from (2) could be true, supported by that law, but if *y* is a plastic object then the antecedent of (2) is not fulfilled and so the statement obtainable from (2) is void. The point here is that the clause "*x* is a metal bar" expresses a condition of application of that formulation of the law of thermal expansion. It is fair to say that if that condition is not fulfilled then the law simply do not apply.

The invariance requirement that Woodward asks for the laws consists in that they "would remain true across many changes in initial conditions." (1992, p. 213, ft. 3). In the former example, the values of L_0 and T_0 express the initial conditions and $L = kL_0T$ is the nomic equation. So, $L = kL_0T$ has to remain true if someone or nature intervenes to change the (kinds of) initial conditions. John Carroll mentions the case where someone is hammering on the ends of a metal bar, granted that it is under a process of heating via another source (2010, p. 14). This is indeed a change, an intervention, in the (kinds of) initial conditions, however, in this case the prior nomic equation does not applies by itsel*f*. It becomes necessary to take into account also the effect in the bar produced by the hammering. Some philosophers concluded from this that such laws are not exceptionless generalizations or that it is required to add some proviso or a ceteris paribus clause to them. Still, the invariance of the holding of that law is preserved when such possible alterations of the system are not introduced.

Another example is about Newton´s gravitational law $F = Gmm'/r^2$ which does not hold by *itself* under the possible circumstances of the influence of the charge of the two bodies involved. With respect to the law of gravitation, Cartwright makes explicit a clause which is not expressed in its mathematical formulation. She says that properly this law asserts: "*If* there are no forces other than gravitational forces at work, *then* two bodies exert a force between each other which varies inversely as the square of the distance between them, and varies directly as the product of their masses." (1983, p. 58). Electricity, like gravity, exerts forces inversely as the square of the distance between charged bodies but, as Feynman points out, the difference in the strength of the electrical and gravitational forces is tremendous, electricity is so much more powerful that gravity. However, also matters how much charge and how much mass the bodies involved have. (cf. 1965, pp. 30-31). In Newtonian celestial mechanics where the bodies have enormous masses the charge force, if any, is negligible, and Newton´s gravitational principle applies by itself. In the atomic world, where particles, say electrons, exert a stronger charge force than gravity force, the latter is negligible.[12] In contrast, on classical particles at human scale, as Cartwright argues: "It is not true that for *any* two bodies the force between them is given by the law of gravitation. Some bodies are charged bodies, and the force between them is not Gmm'/r^2. Rather it is some resultant of this force with the electric force to which Feynman refers." (1983, p. 57). Thus, in the latter cases both the law of gravitation and Coulomb´s law have to be applied in order to determinate the total force.

Therefore, if someone intervenes in a classical system bringing in an external stress on a metal bar with a hammer, or nature intervenes in a two bodies system bringing in a charge force, then in both cases the conditions of the original systems are altered in such a way that we obtain other kinds of systems. Then, of course, it becomes necessary the concurrence of other laws in order to give account of the processes suffered by the modified systems. Nevertheless, the invariances of the laws —the law of thermal expansion and Newton´s law of gravitation— are preserved when such kinds of

[12] Feynman remarks that "There is no Quantum Theory of Gravity today." (1965, p. 33).

interventions are absent. Generally, physical laws prescribe the (kinds of) processes that are possible to happen within some domain (nonetheless, we must not overlook the mentioned abstract and idealized character of fundamental laws). The fair claim is that a law holds invariantly within its domain under certain specific conditions, but beyond of it the law could lose its invariance.

What becomes relevant about counterfactual statements with respect to the lawfulness character of universal statements or invariant generalization is rather the question whether some of the principles of symmetry are counterfactual in character. This calls our attention not to the laws of evolution in physics but instead to the principles of symmetry that hold at theoretical level, at the level of physical equations, which do not have instances in the physical world. The principles that refer to paper and pencil operations, to transformations of equations, like the said reversal of time.

A point of interest is that we can consider that physical laws are universal statements because they hold invariantly under transformations of the physical systems involved as translation in space, translation in time, and replacement of atoms by identical atoms. This means that laws of physics are independent of local places, present times, and particular objects in such a way that we can say that they are universal in a pertinent sense which have to do more with the physical symmetries than with the possibility that they would lose invariance under special changes or interventions beyond their intended domain of application.

Considering laws as explanatory generalizations, Woodward provides a general form of counterfactual statements which laws must support in order to count as invariant generalizations, as follow: (3) "If the value assigned by the variable X to o were to be changed via an intervention (e. g., $X(o) = 0$ to $X(o) = 1$), then the value assigned by Y to o would change in some way predicted by the generalization.", where o stands for a physical object (2003, p. 281). If X and Y stand for state variables of physical systems (e. g., temperature and length), we obtain from the former example that if hammering the extremes of the metal bar altered the value of T then the value of L would change in a way that is not predicted by the thermal expansion law by itself. This result would be considered as a counterexample to (3), however Woodward thought that it is consistent with his invariance-based approach because laws are invariant in different degrees or, better, they can be invariant under a class of interventions or changes and not under any other (2003, p. 284). Thus, (3) is rather a criterion to determine the extent in which a given law, or a general statement, is invariant. Indeed, Woodward purpose is to provide a formulation of the sorts of counterfactuals that would be supported by a law and to find the whole class of counterfactuals that together give us the range of possible circumstances over which the law or generalization is invariant (2003, p. 284). Even he included within such a class of counterfactuals, statements about mental experiments as those that some relativists do in order to contemplate the range of changes over which the field equations of general theory of relativity could be invariant, e. g., worlds with diverse mass distributions extremely different from that in our world (2003, p. 284).

Apart from such interesting mental experiments, I consider that what matters for supporting that laws are invariant generalizations, as Woodward demands, is to find the physical domain on which a law does not break down in the actual world. The changes that are significant to the invariance character of laws are those designed by the principles of symmetry, which are expressed in universal terms like the principles of relativity of Galileo and Einstein. It is very significant for our issue here —the invariance of the holding of physical laws— to know that under a transformation, i. e. reflection of space, laws could collapse, that they could lose symmetry. I guess, that the inverse of time operation is not the sort of change of physical systems that Woodward has in mind, however, this is the sort of transformation which is relevant for stablishing the invariance of the laws and it is indeed a counterfactual principle.

Furthermore, some principles of symmetry are contrary to the facts in character. As we already pointed out, principles about exchange of particles by their antiparticles, reflection of space, and reversal of time, are transformations which are, in some pertinent physical sense, impossible to perform. They are in a relevant and strong sense counterfactual statements. Still, these principles of symmetry, at a high theoretical level, assert the invariance of the holding of some physical laws.

I think that in order to recognize the nomic character of universal statements it is more significant to find out the groups of transformations under which laws are invariants than to establish the class of possible interventions, contrary to the actual facts, under which the laws would be invariant. Though both outlooks do not exclude each other, and the tasks could be complementary.

To sum up, Woodward´s thesis about the relationship between contrafactual statements and laws as invariant generalizations is plausible at the level of laws of evolution but not so at the level of principles of symmetry because some of such principles are contrafactual statements themselves.

Final remarks

We have exposed a number of claims about the connections among invariance, symmetry and lawfulness. Let us now make a recapitulation of the most important ones (in order of appearance except the last one). First, a general claim concerning the symmetry of laws, extracted from physics, is that if a law holds on a kind of processes, it holds invariantly under a specific group of transformations of such processes. Second, it is feasible and useful to make a distinction between two levels of invariance with respect to the laws of physics: one about the manner in which physical systems change according to these laws, whereas the other concerns the symmetry of the laws themselves. On the former level the claim is that the laws of evolution hold invariably within their intended domains of application. On the latter level the claim is that these laws are invariant under specific groups of transformations. Besides, related to the prior first level we have suggested the methodological idea that physicists pursue to extend the actual scope of the holding of the laws within the intended domain of application of a theory, and with respect to the former second level we can add that physicists try to find out where and how symmetries and parities break down. Third, our main claim consists in that the nomic status of the laws of physics resides in their invariance at the two levels already mentioned, which are linked with the principles of conservation —by which one can differentiate such laws from accidental true generalizations without appeal to any kind of necessity. Fourth, borrowing ideas from Kuhn, Moulines, Cartwright and Woodward, we point out that fundamental laws, like Newton´s second law and Schrödinger´s equation, are schemas of statements rather than statements, and that for this reason they are not candidates to bear truth values —although under certain interpretations they could be satisfied in abstract and idealized models. Fifth, regarding the issue of the relationship of laws with counterfactuals we remark that at high abstract level some principles of symmetry are counterfactual in character, which involves that the invariances associated with such principles have only a theoretical significance. Sixth, we can say, as a species of corollary from all the previous, that the traditional philosophical thesis about scientific laws as true and necessary universal statements has no grounding on a number of matters in the field of physics, neither the claim that laws are true statements nor hence the claim that they have a necessary character.

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