

AGAINST PHILO'S INTERPRETATION OF THE CONDITIONAL: THE CASE OF ARISTOTLE'S THESIS

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Resumen

Existe una tesis aristotélica que puede considerarse controvertida. Es la tesis relativa a un condicional negado con solo una variable proposicional y en el que, además, una de sus cláusulas también está negada. Aunque la tesis no es una tautología, las personas tienden a aceptarla como verdadera. El enfoque de Pfeifer puede explicar este hecho. No obstante, yo intento mostrar que este problema se puede explicar igualmente desde otros marcos alternativos, en concreto, desde el de la teoría de los modelos mentales, el de López-Astorga basado en el fenómeno lingüístico de la perfección del condicional y el de la teoría de la lógica mental. Del mismo modo, señalo las dificultades que, con respecto a la tesis de Aristóteles, tienen la teoría de los modelos mentales y la propuesta de López-Astorga, y concluyo que la explicación de la teoría de la lógica mental es la alternativa más fuerte al planteamiento de Pfeifer y que lo que es obvio es que el condicional no debería interpretarse de modo material.

Palabras clave: condicional; perfección del condicional; lógica mental; modelos mentales; lógica probabilística.

Abstract

There is an Aristotelian thesis that can be considered controversial. That is the thesis related to a denied conditional with only one propositional variable and in which, in addition, one of its clauses is also denied. While the thesis is not a tautology, people tend to accept it as true. Pfeifer's approach can account for this fact. However, I try to show that this problem can also be explained from other alternative frameworks, in particular, from that

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of the mental models theory, that of López-Astorga based on the pragmatic phenomenon of conditional perfection, and that of the mental logic theory. Likewise, I indicate the difficulties regarding Aristotle's thesis of the mental models theory and López-Astorga's proposal, and conclude that the account of the mental logic theory is the strongest alternative to Pfeifer's explanation and that what is clearly obvious is that the conditional should not be materially interpreted.

Keywords: conditional; conditional perfection; mental logic; mental models; probabilistic logic.

Introduction¹

In cognitive science field, it has been noted that certain Aristotle's thesis is polemical. There are two versions of that thesis:

$$(V1) \neg(\neg x \rightarrow x)$$

$$(V2) \neg(x \rightarrow \neg x)$$

Where '¬' is denial and '→' refers to the conditional.

As it is well known, that thesis is to be found in *Ἀναλυτικὰ Πρότερα* (*Analytica Priora*) 57b14, where it is said that, given something, its denial is not possible, and it is a very important element of the so-called 'connexive logic', that is, the non-classical logic inspired by the requirement of a connection between the propositions claimed by Chrysippus of Soli (see, e.g., McCall, 1975). However, what is important for the aims of this paper is that, as commented by Pfeifer (2012), people tend to state that the mentioned versions of the thesis are true. Nevertheless, they are not tautologies in standard classical logic, but contingent formulae. Indeed, classical logic assumes the interpretation of the conditional proposed by Philo of Megara, i.e., the material interpretation of the conditional, and, under that interpretation, there are cases in which V1 and V2 are false.

According to the material interpretation of the conditional, as it is also well known, a particular conditional can only be false when its antecedent is true and its consequent is false. In this way, if we consider that 'v' refers to the truth value of the formula that follows between brackets, that '1' denotes truth, and that '0' means falsehood, it can be said that:

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$v(x \rightarrow y) = 1$ when:

i) $v(x) = 1$ and $v(y) = 1$

ii) $v(x) = 0$ and $v(y) = 1$

iii) $v(x) = 0$ and $v(y) = 0$

And that:

$v(x \rightarrow y) = 0$ when:

i) $v(x) = 1$ and $v(y) = 0$

Therefore, if the conditional is materially interpreted, it is evident that there is a possibility in which $v(V1) = 0$. That possibility is the previous case ii), i.e., the scenario in which $v(\neg x) = 0$ and $v(x) = 1$. In that scenario, $v(\neg x \rightarrow x) = 1$ and hence $v[\neg(\neg x \rightarrow x)] = 0$.

Likewise, there is also a situation in which $v(V2) = 0$. It corresponds to ii) too and is the case in which $v(x) = 0$ and $v(\neg x) = 1$. In that case, $v(x \rightarrow \neg x) = 1$ and hence $v[\neg(x \rightarrow \neg x)] = 0$.

Pfeifer (2012) presents an interesting and clear account of this fact. The main idea of that account seems to be that Philo's interpretation should not be assumed and that, therefore, the conditional should not be considered as material. According to Pfeifer, conditionals must be interpreted as conditional events and people reason about conditionals thinking about the probability of the consequent given the antecedent, i.e., in formal terms, thinking about $P(y|x)$. It is obvious that, if this thesis is correct, it can be explained why reasoners tend to consider V1 and V2 to be true. Indeed, the probability of x cannot be very high if $\neg x$ is true, and, likewise, the probability of $\neg x$ cannot be very high if x is true. Furthermore, the relationships between Pfeifer's (2012) proposal and the general theses provided by the proponents of the so-called probability logic (e.g., Adams, 1998; Adams & Levine, 1975; Oaksford & Chater, 2007, 2009; as far as this point is concerned, see also Pfeifer's, 2015, paper) are evident. Following this later logic, a very important idea, which is also adopted by Pfeifer, is that individuals do not usually consider the scenarios in which the antecedent of a conditional is false. This idea, as shown below, is very relevant for the aims of this paper as well. But, for now, perhaps it is enough to state that Pfeifer (2012) carried out different experiments in order to prove that this argument is correct, and that his results, certainly, seemed to demonstrate that.

In this way, it must be said that there is not doubt that Pfeifer's (2012) experimental results are consistent with the idea that people interpret conditionals as conditional events. Nonetheless, I think that Pfeifer's (2012) approach is not the only framework that can explain why people often

accept propositions such as V1 and V2 and tend to reject propositions such as $\neg x \rightarrow x$ and $x \rightarrow \neg x$. In particular, in my opinion, there are at least three more possible approaches: that based on the mental models theory (from now on, M-Mt), that based on the phenomenon of conditional perfection, and that based on the mental logic theory (from now on, M-Lt). Of course, other frameworks could be considered, including those of the social contracts theory (e.g., Cosmides, 1989; Fiddick, 2004; Fiddick & Erlich, 2010) and the pragmatic schemata theory (e.g., Cheng & Holyoak, 1985, 1989). However, I will not take them into account for two reasons: Firstly, the goals of this paper are very modest and it is not intended to review all of the theories trying to explain aspects of human reasoning. Secondly, many of those theories are focused on so specific domains of reasoning that are not clearly relevant here, since I will address a problem related to the general abstract reasoning ability.

Thus, speaking about the accounts that I will review again, it can be said that, maybe, as I will expose, the first two frameworks (that of M-Mt and that of conditional perfection) have some difficulties that must be solved before they are definitely assumed. However, as I will also show, that of M-Lt appears to be strong enough to be taken into account and considered as a clear alternative to Pfeifer's (2012) probabilistic explanation. Thus, in the next sections, I will argue in favor of these ideas. I will start by analyzing the account that can be offered by M-Mt and by indicating what its problems can be. Then I will comment López-Astorga's (2013) explanation (based on conditional perfection) and the difficulties that it also needs to solve. Finally, I will describe how M-Lt can account for this problem and why it is the best alternative to the thesis that conditionals are interpreted as conditional events.

Aristotle's thesis and M-Mt

M-Mt is a theory explained in details in different works (for example, Byrne & Johnson-Laird, 2009; Johnson-Laird, 2010, 2012; Johnson-Laird, Byrne, Girotto, 2009; Khemlani & Johnson-Laird, 2009; Khemlani, Orenes, & Johnson-Laird, 2012, 2014; Orenes & Johnson-Laird, 2012). According to it, people reason by analyzing the different combinations of possibilities, i.e., the different models, which can be attributed to propositions. As far as the conditional is concerned, M-Mt claims that a proposition such as $p \rightarrow q$ has an initial mental model:

$$p / q$$

That is, a model in which both the antecedent and the consequent are true.

Thus, the formula that is within the parentheses in V1 would refer to this model:

$$\neg x / x$$

Given that this is an impossible scenario (x and $\neg x$ cannot be true at the same time), following M-Mt, it is obvious why people tend to reject a formula such as $\neg x \rightarrow x$ and to admit a formula such as V1.

Something similar can be said about V2. In this case, the formula within brackets would have this mental model:

$$x / \neg x$$

Which, again, describes an impossible situation. Therefore, it is also clear, according to M-Mt, why individuals tend not to accept a formula with a formal structure such as $x \rightarrow \neg x$ and to admit a formula such as V2.

But, in my view, M-Mt has a problem regarding this issue. This theory states that there are not only mental models, but also 'fully explicit models'. The fully explicit models are all the combinations that really correspond to a proposition and, often, they can only be identified if reasoners make certain efforts. In particular, the fully explicit models of the conditional are:

$$\begin{array}{l} p / q \\ \neg p / q \\ \neg p / \neg q \end{array}$$

In this way, the fully explicit models of $\neg x \rightarrow x$ are:

$$\begin{array}{l} \neg x / x \\ x / x \\ x / \neg x \end{array}$$

Obviously, the first ($\neg x$ and x) and the third (x and $\neg x$) models must be rejected, since they describe inconsistent scenarios. But the second one (x and x , i.e., x) is absolutely possible. This means that, based on M-Mt, it can be said that, if an individual recovers the fully explicit models of $\neg x \rightarrow x$, that individual should accept that proposition as possible and not consider V1 to be always and necessarily true.

The same can be stated on $x \rightarrow \neg x$. Its fully explicit models are:

$x / \neg x$
 $\neg x / \neg x$
 $\neg x / x$

Again, the second model ($\neg x$ and $\neg x$, i.e., $\neg x$) is perfectly possible. So, it can be thought that, if its fully explicit models are truly detected, $x \rightarrow \neg x$ cannot be considered to be absolutely false, and that, likewise, V2 cannot be considered to be absolutely true.

Thus, although M-Mt can explain the difficulties of Aristotle's thesis, it needs to clarify an important problem: when and under what conditions people do not take only the mental models into account, i.e., when and under what conditions individuals tend to recover the fully explicit models. Pfeifer's (2012) experiments included both abstract tasks and tasks with thematic content, and I think that it is legitimate and justified to assume that, in some of the tasks with thematic content, the participants were likely to detect all of the fully explicit models of the propositions. Furthermore, another interesting point here is that the two models that are added by the fully explicit models set are precisely models in which the antecedent of the conditional is denied. Maybe this could also be interpreted even as an additional support not to the mental models theory, but to the general framework of the probability logic, in which, as said, people do not use to consider scenarios in which the antecedent of a conditional is negated.

On the other hand, Khemlani et al. (2012) proposed an extension of M-Mt related to the denial of propositions. According to that extension, the models of a denied proposition are the complement of the models of that same proposition when it is not denied. In this way, the only model of V1 is:

$\neg x / \neg x$

And that of V2:

x / x

This means that, given $\neg x \rightarrow x$ and V1 –or $x \rightarrow \neg x$ and V2–, without further information, people cannot know which of those propositions is really the true one, because both of them have possible models. Therefore, it can be stated that, from the M-Mt point of view, if the fully explicit models are recovered, both $\neg x \rightarrow x$ and V1 –as well as both $x \rightarrow \neg x$ and V2– are indeterminate formulae, and it cannot be predicted that people will reject $\neg x \rightarrow x$ and $x \rightarrow \neg x$, and accept V1 and V2. Undoubtedly, M-Mt can be an alternative to Pfeifer's (2012) probabilistic approach. It only requires assuming that, when people reason about propositions with structures

akin to the previous ones, they do not identify the fully explicit models. Nevertheless, as said, it needs to give a better explanation of this point and a clearer account of why the fully explicit models are only recovered in certain occasions –and they are not in other circumstances. Furthermore, Khemlani et al.'s (2012) extension also has certain difficulties. According to it, V1 and V2 have possible models, but they are not always true (V1 is not true if x is true and V2 is not true if $\neg x$ is true).

Aristotle's thesis and conditional perfection

Another proposal can be that presented by López-Astorga (2013). Based on works such as Auwera (1997), Horn (2000), and Moldovan (2009), he argues that what happens when people think about V1 and V2 is that they perfect the conditionals included in those formulae. As it is well known, conditional perfection consists of considering a particular conditional to be a biconditional. This pragmatic phenomenon occurs when reasoners interpret that the antecedent of the conditional is not only a sufficient condition of the consequent, but also that it is a necessary condition of it (or, if preferred, when reasoners think that the consequent is a sufficient condition of the antecedent too).

López-Astorga (2013) is aware that the frameworks presented by Auwera (1997), Horn (2000), and Moldovan (2009) are not exactly the same, but, from them, he states that conditional perfection happens when it is hard to find alternative antecedents for a particular conditional. In those cases, people can assume that the antecedent expressed in the proposition is the only possible antecedent, and that the proposition hence does not only mean $p \rightarrow q$, but also $q \rightarrow p$.

Thus, the idea is that people do not interpret V1 and V2 literally, but as follows:

$$(V1b) \neg(\neg x \leftrightarrow x)$$

$$(V2b) \neg(x \leftrightarrow \neg x)$$

Given that $v(x \leftrightarrow y) = 1$ when:

$$i) \quad v(x) = 1 \text{ and } v(y) = 1$$

$$ii) \quad v(x) = 0 \text{ and } v(y) = 0$$

And that $v(x \leftrightarrow y) = 0$ when:

$$i) \quad v(x) = 1 \text{ and } v(y) = 0$$

$$ii) \quad v(x) = 0 \text{ and } v(y) = 1$$

It can be said that $\neg x \leftrightarrow x$ is a contradiction, since neither it is possible a scenario in which $v(x) = 1$ and $v(\neg x) = 1$ at the same time nor it is possible a scenario in which $v(x) = 0$ and $v(\neg x) = 0$ at the same time. Therefore, V1b is a tautology and it is clear why people tend to accept V1 –or V1b– as absolutely true.

In the same way, and for the same reasons, $x \leftrightarrow \neg x$ is a contradiction too and V2b is a tautology as well. So, it is also evident why people tend to consider V2 –or V2b– as undoubtedly true.

López-Astorga (2013) holds that Pfeifer's (2012) experiments include tasks with a very poor context and that is the reason why people tend to think that the conditionals used in those tasks do not have possible alternative antecedents. If the texts of the instructions of Pfeifer's experiments are taken into account, this idea can be convincing. For example, in some experimental conditions, Pfeifer (2012) describes a scenario with two animals –a dog and a cat– and one of those animals corresponds to x and the other one to $\neg x$ in propositions with the formal structure of V1 and V2. Given that no more animals are indicated, indeed, it can be thought that the participants could not imagine alternative antecedents for those propositions.

However, López-Astorga's (2013) account has an inconvenience too. It does not specify which its general framework is. Because López-Astorga refers to the logical concepts 'contradiction' and 'tautology', it seems that his general framework is classical logic. Nevertheless, as it is well known, the thesis that human mind follows the requirements of standard propositional logic is problematic. The literature of cognitive science shows that people do not often reason in accordance with systems such as that of Gentzen (1935). A very illustrative case in this way can be, for example, that of the disjunction introduction rule. As it is also well known, the schemata corresponding to this rule are the following:

$x // \text{Ergo } x \vee y$
 $y // \text{Ergo } x \vee y$

Where 'v' denotes disjunction.

In classical logic, if $v(x) = 1$, then necessarily $v(x \vee y) = 1$. Likewise, if $v(y) = 1$, then necessarily $v(x \vee y) = 1$. Besides, the two previous schemata are valid rules in standard propositional calculus. Nonetheless, several works (e.g., Orenes & Johnson-Laird, 2012; Braine & O'Brien, 1998) reveal that people do not often use these schemata. Thus, cases such as this one can lead one to think that López-Astorga's (2013) explanation needs to be reviewed, or at least that it needs to clarify what it is based on, i.e., which its general theory or framework is. A possible solution could be that

López-Astorga's (2013) proposal is not based on classical logic, but, for example, on a theory such as M-Lt. Nevertheless, as I will show in the next section, if M-Lt is assumed, it is not necessary to suppose that people tend to perfect propositions with structures akin to V1 and V2. Furthermore, the advantage of López-Astorga's account over frameworks such as that of the probability logic is not clear, since the latter can explain not only the difficulties related to Aristotle's thesis, but also many more problems of cognitive science today.

Aristotle's thesis and M-Lt

M-Lt (e.g., Braine & O'Brien, 1998; O'Brien, 2009; O'Brien & Manfrinati, 2010) is a theory that proposes that human mind works following syntactic or formal rules. However, although all of the formal schemata that it admits are rules that hold in standard propositional calculus, not every rule of this calculus is accepted by M-Lt (for example, it does not admit the disjunction introduction rule). In this way, M-Lt is different to classical logic in many respects, but, as far as the aims of this paper are concerned, it is only important to mention two differences. Firstly, it rejects Philo's interpretation of the conditional, i.e., the material interpretation of the conditional. As I understand it, based on this theory, a proposition such as $x \rightarrow y$ only means that, if x is true, y must be true as well. This is because, according to M-Lt, Chrysippus' Modus Ponens ($x \rightarrow y, x // \text{Ergo } y$) is a valid rule (in particular, it is a 'Core Schema'), but people do not reason by using truth tables and they hence do not think about situations in which, for example, $v(x) = 0$ and $v(x \rightarrow y) = 1$. This is very important, since it is a point that Pfeifer's (2012) approach and M-Lt seem to share.

On the other hand, contradictions do not mean the same in standard propositional calculus and in mental logic. In standard propositional calculus, a contradiction allows deriving any formula, by Ex Contradictione Quodlibet principle. Nevertheless, in mental logic, a contradiction only indicates that at least one of the assumptions of the inference is not true, i.e., contradictions only play a role in Reductio ad Absurdum processes. So, the difference between Ex Contradictione Quodlibet and Reductio ad Absurdum is essential in mental logic. In this logic, a contradiction such as $x \cdot \neg x$ (where ' \cdot ' means conjunction) does not enable to suppose any formula in order to demonstrate its denial (i.e., to suppose y and, by virtue of $x \cdot \neg x$ and Ex Contradictione Quodlibet principle, to conclude $\neg y$). According to M-Lt, people only suppose facts that are possible and, if an individual

knows that a fact y is likely to be false, probably, that individual will not suppose y . As mentioned, a contradiction such as $x \cdot \neg x$ only reveals that at least one of the premises assumed to be true is not really true.

That said, we can think about an inference such as this one:

- (1) $\neg x \rightarrow x$ (premise)
- (2) $\neg x$ (premise)
- (3) x (MP 1, 2)
- (4) $x \cdot \neg x$ (\cdot I 2, 3)

Where 'MP' means 'Modus Ponens' and ' \cdot ' stands for the conjunction introduction rule ($x, y // \text{Ergo } x \cdot y$), which is also admitted by M-Lt (in fact, in this theory, it is a 'Feeder Schema').

The incompatibility found in (4) shows that at least one of the premises \neg (1) or (2)- is false. Obviously, it can be thought that the wrong premise is (2). Nonetheless, if we remember that M-Lt rejects Philo's interpretation (and hence that $\neg x \rightarrow x$ only makes sense if x can be deduced from $\neg x$), it is clear that (1) cannot be accepted. The argument is that $\neg x \rightarrow x$ cannot be true because, if $\neg x$ were true, an incompatibility would be found.

This is the argument for V1, but the inference corresponding to V2 would not be very different:

- (1) $x \rightarrow \neg x$ (premise)
- (2) x (premise)
- (3) $\neg x$ (MP 1, 2)
- (4) $x \cdot \neg x$ (\cdot I 2, 3)

Again, this inference represents a situation in which, given (2), the contradiction in (4) leads to reject $x \rightarrow \neg x$.

So, from M-Lt, it is evident why people tend to reject propositions such as $\neg x \rightarrow x$ and $x \rightarrow \neg x$, and to consider propositions such as V1 and V2 to be true. The advantage of this last account is that, unlike that of M-Mt and that based on conditional perfection, it does not have any aspect that needs to be clarified. Therefore, it can be considered to be, at the moment, the strongest alternative to Pfeifer's (2012) approach.

Conclusions

It is clear that Pfeifer's (2012) probabilistic framework can explain the phenomena and difficulties linked to Aristotle's thesis, and this paper has not been intended to prove otherwise. My only aim has been to show that, given that there are alternative explanations of the problem that Pfeifer

(2012) analyzes, if people interpret conditionals as conditional events, that needs to be demonstrated by means of more conclusive evidence.

In this way, I have presented the accounts of three alternative approaches. As it can be noted above, the first two alternatives have problems that require to be solved. M-Mt is a very relevant theory and its explanation is very interesting. However, it is necessary that it indicates the exact factors that can lead an individual to consider the fully explicit models and the complements of propositions when the latter are denied, i.e., that it indicates when individuals only pay attention to the mental models of propositions and when they can identify all of the models.

López-Astorga's (2013) account is also suggestive. The idea that a poor context can cause a conditional to be perfected is plausible and is supported by the literature. Nevertheless, the difficulty here is that it seems that the general framework assumed by López-Astorga is classical logic, and it is highly questionable that human reasoning follows the requirements of standard logic.

Thus, the hypothesis of a mental logic different from classical logic appears to be more feasible. Nonetheless, if a mental logic such as that proposed by M-Lt is accepted, as shown, it is not necessary to suppose that people tend to perfect the conditionals with formal structures similar to those of V1 and V2. The only assumption that is needed is that Philo's interpretation of conditionals is not reasoner's usual interpretation of them. So, it can be stated that M-Lt is a solid framework alternative to that of Pfeifer (2012) and that, therefore, the discussion is open yet.

Of course, it can be thought that what must be done is to look for experimental situations for which the predictions derived from Pfeifer's probabilistic approach and from M-Lt are different, since maybe such experimental situations can offer results that reveal which of these two proposals is the actually correct one. In this way, there is no doubt that versions of tasks related to the conditional such as that of Wason (1966, 1968) can be very useful. In any case, at the moment, a point seems obvious: people do not tend to interpret conditionals in a material way. None of these two accounts (that of Pfeifer and that of M-Lt) assumes Philo's interpretation. And this fact is truly relevant because refers to more aspects that both approaches share too. As said, none of them takes into account situations in which the antecedent of the conditional is denied, which seems to be the key and demonstrate that it is worth continuing to analyze, review, and compare these two frameworks. Furthermore, they appear to describe human reasoning better than other theories, at least as far as the problems considered in this paper are concerned.

References

- Adams, E. W. (1998). *A Primer of Probability Logic*. Stanford, CA: Center for the Study of Language and Information publications.
- Adams, E. W. & Levine, H. P. (1975). On the uncertainties transmitted from premises to conclusions in deductive inferences. *Synthese*, 30, 429-460.
<http://dx.doi.org/10.1007/BF00485053>
- Auwers, J. van der (1997). Pragmatics in the last quarter century: The case of conditional perfection. *Journal of Pragmatics*, 27, 261-274.
[http://dx.doi.org/10.1016/S0378-2166\(96\)00058-6](http://dx.doi.org/10.1016/S0378-2166(96)00058-6)
- Braine, M. D. S. & O'Brien, D. P. (Eds.) (1998). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Byrne, R. M. J. & Johnson-Laird, P. N. (2009). 'If' and the problems of conditional reasoning. *Trends In Cognitive Science*, 13(7), 282-287.
<http://dx.doi.org/10.1016/j.tics.2009.04.003>
- Cheng, P. & Holyoak, K. (1985). Pragmatic reasoning schemas. *Cognitive Psychology*, 17, 391-416.
[http://dx.doi.org/10.1016/0010-0285\(85\)90014-3](http://dx.doi.org/10.1016/0010-0285(85)90014-3)
- (1989). On the natural selection of reasoning theories. *Cognition*, 33, 285-313.
[http://dx.doi.org/10.1016/0010-0277\(89\)90031-0](http://dx.doi.org/10.1016/0010-0277(89)90031-0)
- Cosmides, L. (1989). The logic of social exchange: Has natural selection shaped how humans reason? Studies with the Wason selection task. *Cognition*, 31, 187-276.
[http://dx.doi.org/10.1016/0010-0277\(89\)90023-1](http://dx.doi.org/10.1016/0010-0277(89)90023-1)
- Fiddick, L. (2004). Domains of deontic reasoning: Resolving the discrepancy between the cognitive and moral reasoning literatures. *Quarterly Journal of Experimental Psychology*, 57A, 447-474.
<http://dx.doi.org/10.1080/02724980343000332>
- Fiddick, L. & Erlich, N. (2010). Giving it all away: Altruism and answers to the Wason selection task. *Evolution and Human Behavior*, 31, 131-140.
<http://dx.doi.org/10.1016/j.evolhumbehav.2009.08.003>
- Gentzen, G. (1935). Untersuchungen über das Logische Schließen I. *Mathematische Zeitschrift*, 39, 176-210.
<http://dx.doi.org/10.1007/BF01201353>
- Horn, L. R. (2000). From if to iff: Conditional perfection as pragmatic strengthening. *Journal of Pragmatics*, 32, 289-326.
[http://dx.doi.org/10.1016/S0378-2166\(99\)00053-3](http://dx.doi.org/10.1016/S0378-2166(99)00053-3)

- Johnson-Laird, P. N. (2010). Against logical form. *Psychologica Belgica*, 5(3&4), 193-221.
<http://dx.doi.org/10.5334/pb-50-3-4-193>
- (2012). Inference with mental models. In K. J. Holyoak & R. G. Morrison (Eds.), *The Oxford Handbook of Thinking and Reasoning* (pp. 134-145). New York, NY: Oxford University Press.
<http://dx.doi.org/10.1093/oxfordhb/9780199734689.013.0009>
- Johnson-Laird, P. N., Byrne, R. M. J., & Girotto, V. (2009). The mental models theory of conditionals: A reply to Guy Politzer. *Topoi*, 28, 78-80.
<http://dx.doi.org/10.1007/s11245-008-9049-9>
- Khemlani, S. & Johnson-Laird, P. N. (2009). Disjunctive illusory inferences and how to eliminate them. *Memory & Cognition*, 37(5), 615-623.
<http://dx.doi.org/10.3758/MC.37.5.615>
- Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2012). Negations: A theory of its meaning, representation, and use. *Journal of Cognitive Psychology*, 24(5), 541-559.
<http://dx.doi.org/10.1080/20445911.2012.660913>
- (2014). The negation of conjunctions, conditionals, and disjunctions. *Acta Psychologica*, 151, 1-7.
<http://dx.doi.org/10.1016/j.actpsy.2014.05.004>
- López-Astorga, M. (2013). La tesis de Aristóteles y la interpretación bicondicional de los enunciados condicionales. *Alpha*, 37, 237-248.
<http://dx.doi.org/10.4067/S0718-22012013000200017>
- McCall, S. (1975). Connexive implication. In A. R. Anderson & N. D. Belnap (Eds.), *Entailment. The Logic of Relevance and Necessity*. Volume I (434-446). Princeton, NJ: Princeton University Press.
- Moldovan, A. (2009). Pragmatic considerations in the interpretation of denying the antecedent. *Informal Logic*, 29, 309-326.
- Oaksford, M. & Chater, N. (2007). *Bayesian Rationality: The Probabilistic Approach to Human Reasoning*. Oxford, UK: Oxford University Press.
<http://dx.doi.org/10.1093/acprof:oso/9780198524496.001.0001>
- (2009). Précis of Bayesian rationality: The probabilistic approach to human reasoning. *Behavioral and Brain Sciences*, 32, 69-84.
<http://dx.doi.org/10.1017/S0140525X09000284>
- O'Brien, D. P. (2009). Human reasoning requires a mental logic. *Behavioral and Brain Sciences*, 32, 96-97.
<http://dx.doi.org/10.1017/S0140525X09000429>
- O'Brien, D. P. & Manfrinati, A. (2010). The mental logic theory of conditional propositions. In M. Oaksford & N. Chater (Eds.), *Cognition*

- and Conditionals: Probability and Logic in Human Thinking* (pp. 39-54). Oxford, UK: Oxford University Press.
<http://dx.doi.org/10.1093/acprof:oso/9780199233298.003.0002>
- Orenes, I. & Johnson-Laird, P. N. (2012). Logic, models, and paradoxical inferences. *Mind & Language*, 27(4), 357-377.
<http://dx.doi.org/10.1111/j.1468-0017.2012.01448.x>
- Pfeifer, N. (2012). Experiments on Aristotle's thesis: Towards an experimental philosophy of conditionals. *The Monist*, 95(2), 223-240.
<http://dx.doi.org/10.5840/monist201295213>
- (2015). The new psychology of reasoning: A mental probability logical perspective. *Thinking & Reasoning*, 19, 329-345.
<http://dx.doi.org/10.1080/13546783.2013.838189>
- Wason, P. C. (1966). Reasoning. In B. Foss (Comp.), *New Horizons in Psychology* (pp. 135-151). Harmondsworth (Middlesex), UK: Penguin.
- (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology*, 20, 273-281
<http://dx.doi.org/10.1080/14640746808400161>